3 1761 05478843 5

Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/seniorheat00stewuoft

SENIOR HEAT

For the Cambridge Senior Local Examination.

The School Arithmetic (Senior Local Edition). By W. P. WORKMAN. 3s.

Gospel of St. Matthew, with Introduction, Notes, and Maps. By Rev. T. A. WALKER. 1s. 6d.

A Senior Course of English Composition. By E. W. EDMUNDS. 2s. 6d. [*Ready December* 1911.

Senior Geography for 1912. By G. C. FRY. Covering the Syllabus for 1912 and including British Isles, Asia, Africa, and Northern America. With Examination Questions and Coloured Maps. 2s. 6d.

Senior Latin Course. By A. J. F. COLLINS. 3s. 6d.
[*Ready December* 1911.

The exercises may be had separately at 1s.

Selections from Latin Authors (Senior Edition). With Appendix of passages for practice in Unseens. By A. F. WATT and B. J. HAYES. 2s. 6d.

Senior French Course. By Professor WEEKLEY and C. GILLI. 3s. 6d. [*Ready December* 1911.

Senior French Reader. With Notes and Vocabulary. By R. F. JAMES. 2s. 6d. [*Ready December* 1911.

The School Geometry. By W. P. WORKMAN and A. G. CRACKNELL. 3s. 6d.

Also in Two Parts, as follows:—

Part I. Covering Euclid, I., III. (1-34), IV. (1-9). 2s.

Part II. Covering Euclid, II., III. (35-37), IV. (10-16), and VI. 2s.

Senior Chemistry. By G. H. BAILEY and H. W. BAUSOR. 4s. 6d.

Senior Heat. By R. W. STEWART and J. SATTERLY. 3s.

Senior Botany. By Professor F. CAVERS. 4s. 6d.

LONDON: W. B. CLIVE, DRURY LANE, W.C.

Cambridge Senior Local Examination

SENIOR HEAT

BY

R. W. STEWART, D.Sc.

AND

JOHN SATTERLY, D.Sc., M.A.

AUTHORS OF "TEXT-BOOK OF HEAT," "TEXT-BOOK OF LIGHT," ETC.

12944
3/10/13

LONDON: W. B. CLIVE

University Tutorial Press Ltd.

DRURY LANE, W.C.

1911

LIBRARY OF THE UNIVERSITY OF TORONTO

PREFACE.

This book provides a course of theoretical and experimental work on Heat specially suited to modern methods of teaching Science in schools.

The student is encouraged to experiment for himself, so that he may come into actual contact with the facts of the science, and to think for himself so that he may understand and assimilate the theory of the subject. Although the unalloyed heuristic method is recognised to be impracticable, heuristic exercises are included, especially in the earlier chapters; and the greatest care has been taken throughout to secure that correlation between theory and practice which alone enables the student to follow the development of the subject with intelligent interest, and, at the same time, gives him some insight into the methods of scientific investigation.

The leading facts and principles of the subject are deduced from practical work, the greater part of which can be performed by the student himself without the use of elaborate and expensive apparatus. At the beginning of each new subject the experiments are mainly qualitative, and are designed to elucidate the general nature of the phenomena under investigation. Quantitative experiments are introduced later with the object of showing how the general principles of the subject are reduced to definite laws and quantitative statements, capable of exact mathematical expression.

The treatment of the experimental work has received special attention throughout. In the description of each

experiment details and practical hints are given which enable the student to perform the experiment for himself and to obtain a satisfactory result. The student is trained to consider the probable errors that may arise in any particular experiment, and to note that the method used and the arrangement of the apparatus must be adapted to eliminate or minimise these errors.

Where classical experiments are described they are usually associated with simpler experiments which illustrate the methods adopted by the original workers, and are capable themselves of giving sufficiently good results in the laboratory.

The calculations which occupy such an important place in any branch of Physics usually present great difficulties to the elementary student. This is probably due, in some measure, to the abuse of mathematical formulae. The use of formulae is therefore discouraged in the earlier portions of these volumes. The student is at first taught to work as far as possible from first principles, and to express the rules by which he works in words. Later, when it becomes necessary for the student to work quickly and accurately and to set out his work clearly and concisely, the use of formulae is permitted and encouraged.

In this volume on *Heat* a special chapter has been devoted to methods of approximation, so that the student may understand that he must not give more significant figures in a result than are justified by the data, and may know how to avoid unnecessary figures in calculating.

This book is largely the same as *The New Matriculation Heat*, by the same authors, but considerable additions have been made in the two last chapters, so that in its present form it covers all that is required for the Senior Cambridge Local Examinations.

CONTENTS.

CHAPTER I.

PAGE

Introductory 1
Exercises I. 7

CHAPTER II.

The Thermometer 8
Exercises II. 23

CHAPTER III.

Approximations 25
Exercises III. 29

CHAPTER IV.

Expansion of Solids 30
Exercises IV. 49

CHAPTER V.

Expansion of Liquids 52
Exercises V. 73

CHAPTER VI.

Expansion of Gases 76
Exercises VI. 100
Examination Questions 102

CHAPTER VII.

Calorimetry 106
Exercises VII. 112

CHAPTER VIII.

Specific Heat 113
Exercises VIII. 126

CHAPTER IX.

Change of State. Liquefaction and Solidification . . . 130
Exercises IX. 147
Examination Questions 149

CHAPTER X.

PAGE

Properties of Vapour 152
Exercises X. 162

CHAPTER XI.

Change of State. Vaporisation and Condensation . . . 163
Exercises XI. 180

CHAPTER XII.

The Latent Heat of Vaporisation 182
Exercises XII. 189

CHAPTER XIII.

Hygrometry 191
Exercises XIII. 203
Examination Questions 205

CHAPTER XIV.

Conduction of Heat 208
Exercises XIV. 223

CHAPTER XV.

Convection 225
Exercises XV. 231

CHAPTER XVI.

Work and Energy 232
Exercises XVI. 240

CHAPTER XVII.

The Mechanical Equivalent of Heat 242
Exercises XVII. 257

CHAPTER XVIII.

Radiation 259
Exercises XVIII. 283
Examination Questions 287

ANSWERS 291

INDEX 297

HEAT.

CHAPTER I.

INTRODUCTORY.

1. Heat sensations. We distinguish in every-day life between "hot" and "cold" bodies by the sense of touch, and we are familiar, in the same way, with the fact that one body may be "hotter" or "colder" than another, or that the same body may become "hotter" or "colder."

When we touch a substance hotter than the hand we experience the sensation of heat, and we say that heat passes from the substance to the hand. In the same way when we touch a substance colder than the hand we experience the sensation of cold, and we say that heat passes from the hand to the substance. In each case the intensity of the sensation depends upon the extent to which transfer of heat between the hand and the substance takes place, and, as will be learnt later, this depends not only upon how hot or cold the substance is, but also upon the nature of the substance.

It must be noted, however, that sensation is not always a safe guide in determining the relative hotness or coldness of bodies. For example let the right hand be placed in hot water and the left in cold water for a minute or so, and then let both hands be plunged simultaneously into lukewarm water. This water will feel hot to the left hand and cold to the right hand. This shows that in interpreting sensations of heat and cold the previous state of the hand, or other organ of touch, must be considered. Again, in

the hot room of a Turkish bath all the objects in the room are equally hot and hotter than the body. To the sense of touch, however, all objects made of good conducting material, such as metal, appear to be much hotter than objects made of bad conducting material, such as wood or cloth. This, as explained later, is due to the fact that in the case of good conducting materials the transfer of heat to the hand is greater and more rapid than in the case of bad conducting materials. Similarly, on a cold day an iron bar lying out of doors will appear much colder than a broomstick, owing to the fact that heat is transferred more rapidly from the hand to the iron, which is a good conductor, than to the wood, which is a bad conductor. These illustrations show that in interpreting sensations of heat and cold the nature of the substance touched must be considered as well as the state of the organ of touch.

It should be noted that the sensations referred to in this article are those due to actual contact with a hot or cold body.

The sensations of heat and cold experienced in the neighbourhood of hot and cold bodies are usually due to radiation. This subject is dealt with in Chapter XVIII.

2. Temperature. Instead of saying that a body becomes hotter or colder we may say that a body acquires higher or lower degrees of hotness. Degrees of hotness below a certain standard, commonly that of the human body, might perhaps be spoken of as degrees of coldness, but it is more convenient to use one term—hotness—as applicable throughout the whole range of temperature. To speak of degrees of hotness is, however, clumsy and inconvenient. The term **temperature** is therefore used when we deal with the idea of degree in relation to hotness. When a body has acquired any particular degree of hotness it is said to be at a particular temperature, and as a body becomes hotter its temperature is said to rise, and as it cools its temperature is said to fall.

3. Difference of temperature. It is a matter of common experience that when a hot body is placed in contact with a colder one, the hotter body becomes colder and the colder

body becomes hotter. We say in such a case that "heat" has passed from the hot body to the cold one. Thus when a kettle containing cold water is placed on the stove-plate, or on a fire, heat passes from the hot plate or fire to the cold water in the kettle. Also if a cold piece of metal is placed in contact with a hot piece heat passes from the hot piece to the cold piece until the two pieces attain the same temperature. We may therefore define the **difference of temperature** between two bodies A and B as that which is essential for the transfer of heat to take place from A to B or B to A. If two bodies are at different temperatures, then when they are placed in contact heat passes from the body at higher temperature to the body at lower temperature. Or, if two portions of the same body are at different temperatures, heat passes from the portion at higher temperature to the portion at lower temperature. Similarly if two bodies are at the same temperature no heat passes from one to the other when they are placed in contact.

The transfer or exchange of heat between bodies not in contact belongs to the subject of radiation. See Chapter XVIII.

4. Nature of heat. When a substance is heated it is said to gain or absorb **heat**. When it cools it is said to lose or give out **heat**. A question at once arises as to what may be the nature of what is here called **heat**. It is evidently not material, for the mass of a body does not change as it loses or gains heat. The question is specially dealt with in Chapter XVII., but it may be stated here, briefly, that whenever heat is produced energy in some form or other is found to disappear, and, in accordance with the theory of transformation of energy, it is recognised that this energy has been transformed into heat—and that **heat is therefore a form of energy**. Thus when coal burns in air the chemical energy of the products of the combustion is less than that of the original coal and air concerned, and the quantity of chemical energy so lost merely appears as the energy of heat. Similarly when a piece of lead is hammered the energy used by the wielder of the hammer is not lost, but is found as heat in the flattened piece of lead and in the hammer.

Hence, as a body is heated it gains energy in the form of heat, and as it cools it loses energy in the form of heat.

5. General effects of heat. When a body is sufficiently heated or cooled various well marked effects are generally produced. The general character of these effects is the same for most substances. The more important are included under the following heads—

(1) Change of temperature.
(2) Expansion or contraction, *i.e.* change of volume.
(3) Change of state.
(4) Change in physical properties.
(5) Promotion of chemical action.

6. Change of temperature. This is almost axiomatic, for we have assumed that if the temperature of a body is rising it is absorbing heat. It is, however, only true when a body is not changing its state (Art. 8). When water is boiling its temperature remains constant; extra heat given to the water is used up in changing some of the water to steam, and the temperature of the steam is still the same as that of the water.

Again, when water is being frozen heat is abstracted from the water, but the mixture of ice and water still remains at the freezing point, and not until all the water has been frozen is it possible for the ice to get colder than this temperature. When, however, a substance remains in one definite state—solid, liquid, or gaseous—gain of heat produces a rise in temperature, loss of heat produces a fall in temperature. Conversely a rise in temperature indicates a gain in heat, and a fall in temperature a loss of heat.

7. Expansion. It is found that in general when a body is heated it expands, and will, if allowed to cool to its original temperature, gradually contract to its original volume. This is the same whether a body be a solid, a liquid, or a gas. The amount of contraction is different for different bodies; in general, gases expand more than liquids and liquids more than solids for the same rise of temperature.

Exp. 1. Obtain a flat strip of brass about 18 inches long. Screw down one end on a fixed wooden block, B (Fig. 1), allowing the other end to rest freely on a second block, A. If the top of A is not quite even cover it with a piece of finely ground glass or slightly roughened

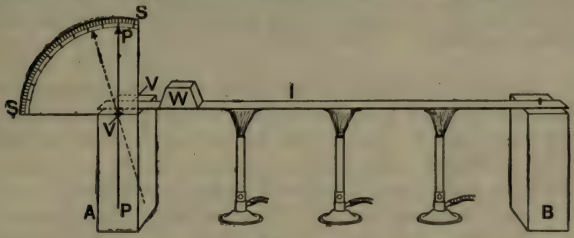


Fig. 1.

metal. Also, if necessary, roughen the under side of the end of the strip. Take a straight short piece of knitting needle, V V, and to one end of it fasten by sealing-wax at right angles to its length a piece of straw, P P, six inches long to act as index. Insert the other end of the needle under the end of the strip. If the weight of the strip is not sufficient to keep the needle and pointer in any position in which they are put, a weight, W, sufficient for this purpose should be laid on the strip. Behind the pointer place a graduated quadrant scale. Now heat the strip by Bunsen burners and observe that the index slowly moves over the graduated scale, finally taking up a position of rest. Now the motion of the pointer tells us that the needle has rotated, and the cause of this rotation is obviously a movement of the rod in the outward direction of its length. This movement is called an expansion. Now remove the burners. As the rod cools the pointer moves back to its former position, showing that as the temperature falls the rod contracts.

Fig. 2.

Fig. 3.

Exp. 2. Fit a 4-oz. round-bottomed flask (Fig. 2) with a cork through which passes the end of a piece of coarse thermometer tubing about 8 or 9 ins. long. Completely fill the flask with alcohol coloured with a little magenta. Then insert the cork so as to make the liquid rise a short distance in the tube. Now place the flask in a beaker of hot water. The liquid will rise in the tube and overflow at the top if allowed to do so.

Exp. 3. Fit a 4-oz. round-bottomed flask (or, better, a bulb with very short neck) with a cork through which a piece of glass *quill* tubing passes nearly to the bottom (Fig. 3). Put in the bottom of the bulb a little coloured alcohol, into which the bottom of the tube is allowed to dip. Now hold the bulb in the hand. The heat thus imparted to the air will cause it to expand and the alcohol will be forced up the tube.

8. Change of state. Bodies may exist in three states—solid, liquid, and gas, and the change from one to the other may always be effected by the application or withdrawal of heat. Thus, for example, if ice is heated it melts into water, and if water is heated sufficiently it boils away and becomes steam. The reverse of the process is the withdrawal of heat from steam, when it condenses to water, and the withdrawal of heat from water, when ice is formed.

9. Change of physical properties. It is found that heating and cooling modify in many ways the properties of matter. Iron heated to redness differs materially from iron at ordinary temperatures; zinc at ordinary temperatures is hard and brittle, but when warmed it is soft and flexible; steel may be softened or tempered by heating and cooling, and so on.

Again, some bodies change colour on being heated: thus mercuric iodide is red at ordinary temperatures, but changes to yellow when heated.

The elasticity and rigidity of solids, the viscosity of liquids, the ease with which a substance conducts electricity, and, in fact, all the characteristic physical properties of matter change with change of temperature.

10. Chemical change. It is well known that heat frequently tends to promote chemical action. Thus coal, wood, sulphur, magnesium wire, and many other substances when heated with free access of air combine vigorously with the oxygen of the air, and we have the phenomenon of *combustion*, which is simply oxidation accompanied by a large output of heat. Also coal, wood, and other compound substances when heated by themselves in a closed space

undergo *decomposition*, and give rise to various products of decomposition; thus coal gives rise to coal-gas and coal-tar.

In fact chemical reactions may be divided into two classes—those which depend upon rise of temperature for their accomplishment and those which occur spontaneously with the evolution of heat.

Again, if calcium carbonate (chalk will do) is heated to a high temperature it decomposes or *dissociates* into calcium oxide and carbonic acid gas. If the products of dissociation are cooled together calcium carbonate is again formed with evolution of heat.

EXERCISES I.

1. Distinguish between heat and temperature.

2. Enumerate the effects of heat.

3. Describe experiments showing that solids, liquids, and gases expand on heating.

CHAPTER II.

THE THERMOMETER.

11. Measurement of temperature. When we have said hitherto that an object was *hot*, we meant that when we touched it with the hand heat flowed from it to the hand; and when another object was said to be *cold*, we meant that when we touched it heat flowed from the hand to it. The *criterion*, therefore, was the temperature of the hand, and our judgments were based on *sensations*. To this method of estimating temperature there are three serious objections:—

(1) Our sensations are not sufficiently delicate. (2) They do not admit of numerical measurement. (3) The criterion (the temperature of the hand) is variable.

It is, therefore, necessary to have some more certain means of comparing temperatures. Instruments called thermometers have been devised for this purpose.

DEFINITION.—**The thermometer is an instrument for comparing temperatures.**

12. Preliminary idea of a thermometer. We saw in Art. 5 that the effects of heat on matter are very diverse. The one which lends itself best to the measure of degree of hotness (or temperature) is that of change of volume (or expansion). So long as no change of state occurs expansion, in most substances, is simultaneous with change of temperature, and change of length or change of volume may therefore be taken to indicate change of temperature. Also, a particular length of any given rod or a particular

volume of any given quantity of liquid corresponds to a particular temperature and may be taken as an indication of that temperature.

Exp. 4. Take a flask and tube as set up in Fig. 2, provide the tube with a graduated paper scale reading upwards, and carry it about to different parts of the building inside and outside. Near the fire it will be observed that the reading is highest. As it is taken to colder parts of the building note that the reading is low and to the hotter parts the reading is high. Suppose now that the reading when in Room A is the same as that when in Room B, what may we say about their temperatures? Proceeding on the above assumptions, obviously they are equal.

With a flask and tube as used in the above experiment the changes in the volume of the liquid are clearly indicated, and with a large bulb and a stem of very narrow bore an instrument capable of indicating very small changes of temperature may be constructed.

In practice a glass, flask, or bulb filled with mercury is found to work most satisfactorily, and such is invariably used if the temperature to be measured is likely to rise nearly as high as that of boiling water. If the indicator be provided with a scale etched on the stem its readings can be more readily noted and compared, and the usefulness of the instrument is greatly increased. The scale adopted may be quite arbitrary. A scale of millimetres, for example, would serve, but it would evidently be an advantage if the scale were arranged to indicate degrees of temperature in accordance with some generally understood and easily realised scale. Such a scale may be obtained as follows: Let the temperature indicator be placed in melting ice, and when the position of the liquid column in the stem is steady, let it be marked by a fine mark on the stem. This mark indicates the temperature of melting ice, which experiment teaches us to be a constant, naturally fixed temperature. Then place the indicator in steam from water boiling under the normal atmospheric pressure, and when the liquid column is again steady let its position be marked. This second mark indicates the temperature at which water boils under the normal atmospheric pressure, and this is again by experiment found to be a constant,

naturally fixed temperature. These two temperatures are generally known as the *freezing point* and *boiling point* of water, and from these two *fixed points* a scale is readily derived.

13. The principle of graduation of a scale of temperature. The range of temperature between the *freezing point* and the *boiling point* may be divided into any number of steps by dividing the volume of the stem into a corresponding number of equal parts. For example, the range of temperature from freezing point to boiling point may be divided into 100 steps or *degrees* by dividing the volume of the bore of the stem between the two marks corresponding to these temperatures into 100 equal parts.

It should be carefully noticed here that nothing is said about dividing the range of *temperature* into 100 *equal* parts. The volume of the stem bore between the two marks is divided into 100 equal parts, but the corresponding steps or degrees of temperature indicated by these divisions may or may not be equal, and are evidently determined by and vary with the expansibility of the liquid used in the indicator. For example if two indicators are accurately made and graduated in the way here described they will usually give two different scales of temperature ; that is, if both were placed together in a bath where the temperature is gradually raised from freezing point to boiling point, it is probable that the temperature readings would agree exactly only at the fixed points. The question that here arises is not which is right, but which is to be adopted as the standard, that is, which is to be set up as "right" by definition and common convention. Hence, if an indicator of this kind is to be used as a standard instrument for general reference, the liquid and the material of the bulb and stem must be specified.

An instrument constructed and graduated on these principles to indicate temperature with reference to an arbitrary selected scale is known as a **thermometer**. Certain substances are better suited than others for the measuring of temperature. A body which changes volume only to a small extent for large variations of temperature is not suitable. Solids are not used as thermometers because their alterations in size even when heated or cooled considerably are minute. Gases change volume very readily when exposed to different temperatures, and for certain

purposes an air thermometer is a useful instrument. Liquids are, however, the best adapted for thermometry.

Of liquids, mercury is the best for all general purposes, because it boils at a very high, and freezes at a very low, temperature. Hence it remains liquid through a long range of temperature. Water has a comparatively small range of temperature between its freezing and boiling points. Alcohol is useful for measuring very low temperatures at which mercury would be frozen, and it is also used in some self-registering minimum thermometers, but it cannot be used for high temperatures because it boils at a lower temperature than the boiling point of water. Mercury has some other advantages. It quickly assumes the temperature of anything in which it is immersed without abstracting much heat from the substance, and it does not soil the glass. Standard thermometers are usually mercury-in-glass thermometers with a scale engraved on the glass stem.

14. Construction and filling of a mercury-in-glass thermometer tube. We now give the usual laboratory method of constructing a rough student's form of thermometer.

A length of thick capillary tubing, having as uniform a bore as can be found, is taken and well washed, first with nitric acid, then with water, and finally dried by a current of hot air. Before proceeding to construct the instrument, it is well to make a preliminary test of the uniformity of the bore of the tube chosen. For this purpose a short thread of mercury about an inch long is introduced by suction into the tube.

With a little management, a thread of the desired length may be got into the bore and moved into any part of the tube. This done, the length of the thread is measured by suitable means in a particular position in the tube; the thread is then moved into some other position, and again measured, and so on. If the length remains constant for different positions it is evident that the bore must be perfectly uniform. This is never found to be the case with any tube, and if the differences in length are very great the tube must be rejected and another tried. When

finally a suitable tube is obtained, one end is heated till melted, and is then blown into a bulb. The size of the bulb is determined by the purpose the thermometer is intended to serve. The larger the bulb the longer will be the degrees of the scale.

In order to introduce mercury into the bulb a funnel-shaped reservoir is made at the top of the tube, or a small funnel is attached to the tube by means of an indiarubber joint (Fig. 4), and more than sufficient mercury to fill the bulb is poured into it. The bulb is gently heated, so that the air in it expands, and some of it escapes by bubbling through the mercury. It is then allowed to cool, and as the air contracts a small quantity of the mercury from the reservoir enters the bulb. This process of alternate heating and cooling is continued until the bulb is about half full of mercury.

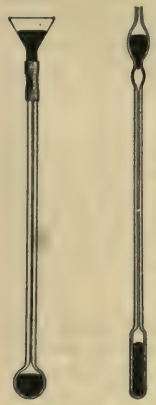


Fig. 4.

The bulb is then gently heated until the mercury boils and its vapour expels the air, which bubbles through the mercury in the upper reservoir. All traces of air and moisture are thus removed, and when the tube is allowed to cool the mercury in it forms a continuous column filling the bulb, stem, and part of the funnel. While still warm the excess of mercury in the last is poured off and the tube is heated at a point just below the funnel and then drawn out so as to greatly constrict the bore. The whole instrument is now heated a little above the highest temperature it is intended to indicate. The

Fig. 5.

mercury now completely fills the stem, and while it does so a blow-pipe flame is brought to bear upon the constriction and the upper part of the tube is drawn off and the thermometer finally closed. The instrument is now ready for graduation, but it is better to keep it from two to three weeks before proceeding with this process, because when glass has been heated to a high temperature it takes a long time (in some cases years) to recover its original volume.

Fig. 5 shows the usual forms of a mercury-in-glass thermometer.

If great accuracy is desired the thermometer should be kept for several months before graduation.

15. Determination of the fixed points of the thermometer scale. As a preliminary to graduation it will be necessary to mark on the stem the position of the thread corresponding to the two standard temperatures referred to above as the freezing point* and boiling point of water.

Fig. 6.

The freezing point. To determine this point the thermometer is placed in a cylindrical vessel † and the reservoir and the lower part of the stem are completely surrounded by melting ice (Fig. 6). When the level of the mercury becomes perfectly stationary a fine mark is made on the stem at the extremity of the column. This is one of the fixed reference points on the scale and indicates the temperature of melting ice.

Boiling point. In order to mark this second point on the thermometer the apparatus shown in Fig. 7 is used. It is made of copper and the steam from water boiling in

* The lower fixed point is always called the *freezing* point, though, in reality, it is the *melting* point of ice, not the freezing point of water. The freezing point of water, in fact, is not constant, but is influenced by several circumstances.

† Sometimes a funnel is recommended for this purpose, but this is unnecessary, for as long as any ice remains in the cylinder the temperature is that of the freezing point.

the lower part circulates round the double casing of the upper part, and finally escapes by a side tube near the bottom of the outer casing. The thermometer is passed through a cork, and placed in the apparatus in the position indicated in the figure. After some time the mercury becomes stationary, and the thermometer is adjusted so that the top of the mercury column is just seen above the cork; its position is then marked, and the second reference point on the scale is thus determined. It indicates the temperature of steam from water boiling under the atmospheric pressure indicated by the barometer at the time of the determination. For the present we shall suppose this to be normal.

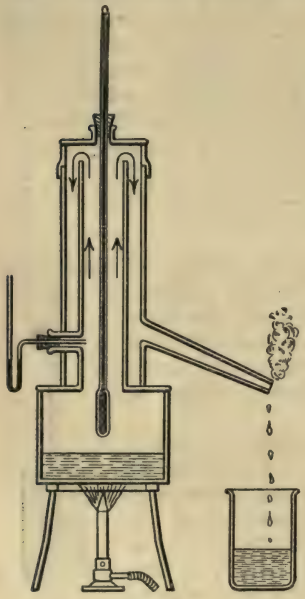


Fig. 7.

If such an apparatus as shown in Fig. 7 is not available—and it is not really necessary, except for the most accurate work—a flask with a long neck may be used. The thermometer is supported by a clamp so that it hangs centrally down the neck of the flask, as much of the thermometer being in the flask as possible, care, however, being taken that the bulb is well above the boiling water. (See Fig. 63 for another form of flask, and Fig. 10 for another form of the apparatus.)

It should be noticed that the thermometer is placed in the steam and not in the boiling water. It has been found that the temperature of boiling water depends on a variety of circumstances, whereas the temperature of the steam depends only on the pressure at which boiling takes place.

16. Graduation of the thermometer. The two reference marks being thus obtained, it remains to graduate the space between them, and to continue the division throughout the whole length of the stem. A zero point is then chosen and the divisions below it are marked negative, and those above positive. There are three scales of graduation now in use; and as we may have temperatures expressed in any one of these scales, it will be necessary to consider them. They are called the Centigrade, Fahrenheit, and Réaumur scales (Fig. 8).

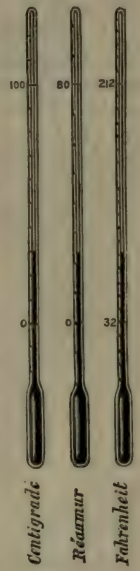


Fig. 8.

For the Centigrade scale we are indebted to Celsius. On it the freezing point is taken as zero and marked 0°; the boiling point is marked 100°. The space between the fixed points is thus divided into 100 equal divisions, called degrees. Graduation on the same scale is extended beyond the reference points; and divisions below zero—*i.e.* below freezing point—are considered negative, and those above positive. This scale is the one most generally used for scientific purposes.

The Fahrenheit scale was introduced by Fahrenheit, of Dantzig, about the year 1714. On it the freezing point is marked 32° and the boiling point 212°, the space between being divided into 180 equal degrees, and the division extended above and below the fixed points. In this method of division the zero is at a point 32° below the freezing point; it is supposed to indicate the lowest temperature which can be got by mixing snow and salt. This scale is very generally used in Great Britain for the purposes of ordinary life, and to some extent for those of science.

On the Réaumur scale the freezing point is taken as zero and marked 0°, and the boiling point is marked 80°. There are thus 80 equal divisions between the fixed points on this scale. It is used in several European countries for medical and domestic purposes, but is never used in England.

A **degree of temperature** on any thermometric scale is the change of temperature corresponding to the expansion of the mercury in the thermometer from one division of the scale to the next.

A particular temperature on any thermometric scale is the temperature indicated by a thermometer when the mercury in the stem stands at a particular division on that scale. For example, the temperature 15° C. is the temperature indicated when the mercury in the stem of a mercury-in-glass thermometer stands at the 15th division above zero on the Centigrade scale.

It is sometimes necessary to convert temperatures expressed in one scale into the corresponding temperatures in one of the other scales. To effect this it is only necessary to remember that the range of temperature from freezing point to boiling point is the same on each scale, and that the freezing point on the Fahrenheit scale is 32 degrees above the zero of that scale. Hence 180 degrees on the Fahrenheit scale are equivalent to 100 degrees on the Centigrade scale and 80 degrees on the Réaumur scale, that is 9 degrees Fahrenheit = 5 degrees Centigrade = 4 degrees Réaumur; and a temperature F on the Fahrenheit scale is (F − 32) degrees above the freezing point. If therefore F, C, and R denote corresponding temperatures on the Fahrenheit, Centigrade, and Réaumur scales we evidently have the relation

$$(F - 32) : C : R :: 9 : 5 : 4 \quad \text{or} \quad \frac{F - 32}{9} = \frac{C}{5} = \frac{R}{4}.$$

Example. If a Fahrenheit and a Centigrade thermometer are placed in the same liquid, and the Fahrenheit instrument reads 68°, what is the reading of the Centigrade instrument?

The liquid is (68 − 32) = 36° F. above freezing point.

Therefore the distance from the freezing point to the head of the mercury column

= $\frac{36}{180}$ (distance from the freezing to the boiling point)

= $\frac{1}{5}$ (distance from the freezing to the boiling point).

Therefore in the Centigrade thermometer

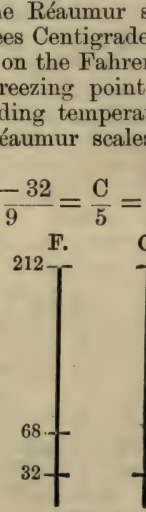


Fig. 9.

also the distance from the freezing point to the head of the mercury column

$= \frac{1}{5}$ (distance from the freezing to the boiling point)

$= \frac{1}{5} \times 100 = 20.$

Therefore the indication on the Centigrade instrument is 20°.

Note.—In working such problems as this it is as well to sketch two thermometers side by side as in Fig. 9 in order to understand clearly that it is the tubes between the freezing and the boiling points which are to be divided proportionately.

Exp. 5. Test the correctness of the freezing-point mark on a thermometer. Chisel off enough ice from a block of ice with a large chisel to fill a beaker. Add a little water and insert the thermometer well into the mixture. Wait till the reading is constant, then note it down. The difference between the reading and the freezing point as marked is the error of the thermometer at this temperature. If the reading is above the freezing point the correction is negative, if below it is positive.

Exp. 6. Test the correctness of the boiling-point mark on a thermometer. A test tube provided with a slit cork and thermometer (Fig. 10), or a flask provided with a side tube and thermometer (Fig. 63), is suitable for this purpose. The thermometer is thrust well down through the cork until the bulb is about an inch or two above the water in the flask. Boil the water for about 5 minutes; the mercury column should then be quite steady, and the position of the top of the column can be read. This will be the boiling point. Correct for pressure if necessary,* and then find the correction to be applied to the thermometer at this temperature.

Fig. 10.

17. Different forms of thermometers. For special purposes various forms of thermometers are used. The more important of these are described below.

* The temperature of boiling depends on the atmospheric pressure. By definition (see Chapter XI.) the temperature of steam from boiling water is 100° C. only when the barometer reads 760 mm. at sea level in latitude 45°. If, therefore, great accuracy is required read the barometer. At sea level in the latitude of London the temperature of boiling is approximately ·0367° C. below or above 100° C. for each millimetre that the barometer (corrected to 0° C.) is below or above 759·6 mm. (See Art. **105** and footnote.)

The Alcohol Thermometer. Alcohol is sometimes used instead of mercury in the construction of thermometers. It has the advantage that it can be employed for very low temperatures : mercury solidifies at — 39° C., whereas alcohol may be exposed to a very much lower temperature without solidifying. It also expands much more than mercury for the same rise of temperature, but the expansion is not uniform with temperature as indicated by a mercury-in-glass thermometer. The expansion increases with the temperature, and for this reason alcohol thermometers are usually graduated by direct comparison with a mercurial thermometer.

18. Maximum and minimum thermometers. It is often necessary to know the highest or lowest temperature reached in a given interval of time. For instance, it is usual in making meteorological observations to record the highest (maximum) temperature attained during the day and the lowest (minimum) temperature reached during the night. For these purposes maximum and minimum thermometers are employed.

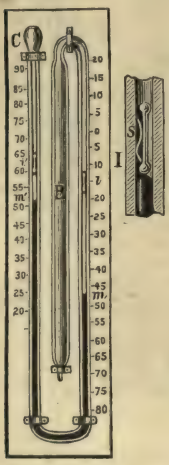


Fig. 11.

Six's maximum and minimum thermometer is one of the oldest of its class, and is at once a maximum and minimum thermometer. Its general plan of construction is shown in Fig. 11. The bulb B and part of the stem down to *m* is filled with alcohol. This is the real thermometric part of the instrument : the column of mercury, *m m'*, which occupies the U-shaped part of the tube merely acts as an index. Above *m'* the tube contains alcohol, as also does the lower half of the bulb C. The upper half of C contains alcohol vapour only, thus leaving room for expansion. As the alcohol in B expands or contracts the extremities, *m* and *m'*, of the mercury

column rise or fall as the case may be, and the extreme point reached in each case is indicated by one end of a light steel index, *i* or *i'* (shown on a larger scale at I). This index is pushed before the mercury column, and is prevented from returning by means of a spring, S, which is just strong enough to hold it in its place. The position of the index *i* evidently indicates the minimum temperature reached, and that of *i'* the maximum temperature. When it is required to set the instrument ready for any observation, these indices are brought into contact with the ends *m* and *m'* of the mercurial column by means of a small magnet. This instrument is largely used by gardeners and nurserymen.

Rutherford's maximum and minimum thermometers are two separate instruments, but are usually mounted on the same frame, as shown in Fig. 12.

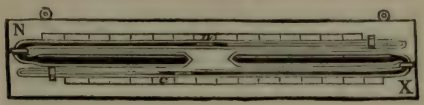


Fig. 12.

The maximum thermometer, X, is a modification of the ordinary mercurial thermometer placed in a horizontal position. The maximum temperature is registered, as in Six's instrument, by means of a steel index, and the method of setting previous to taking an observation is the same. As the thermometer lies horizontally there is no necessity to attach a spring to the index. The maximum temperature will thus be indicated by the position of that end of the index which is nearest the bulb of the thermometer.

The minimum thermometer, N, is an alcohol thermometer, and is the only minimum thermometer in general use. It is placed in a horizontal position, and for registering the minimum temperature there is a small index of glass or enamel, which allows the column of alcohol to expand past it without moving it; but when the alcohol contracts, the index, being wetted by the liquid,

is drawn backwards by the surface film at the extremity of the alcohol column. The minimum temperature is thus indicated by the position of that end of the index which is furthest from the bulb.

The instruments are set by inclining the frame in such a way as to cause the indices to slide down to the ends of the liquid columns.

Fig. 13.

Phillips' maximum thermometer (Fig. 13) is a mercury thermometer with a very fine bore, in which the column of mercury is broken by a small bubble of air which separates a short thread of mercury from the rest of the column. This thread takes the place of the index in Rutherford's thermometer. The instrument, which usually occupies a horizontal position, is set by placing it in a vertical position and gently tapping it. The bubble of air is thus reduced to its smallest dimensions, the gap in the column being about 1/32 inch in length. When the mercury expands the index thread is pushed on so long as expansion continues. When contraction takes place the thread remains stationary, and the graduation is so arranged that the forward end indicates the maximum temperature experienced since the instrument was last set.

The ordinary *clinical thermometer* (Fig. 14) is a form of maximum thermometer. It is constructed on the principle of Phillips' instrument, and is graduated to give a short range of temperature above and below 98·4° F., the normal temperature of the body. If placed in the mouth or arm-pit for a few minutes until it attains a stationary temperature its maximum reading gives the temperature of the body.

Fig. 14.

There is one important form of thermometer which we have not mentioned in this article. It is the *air thermometer*. When we come to consider what should be the properties of a perfect thermometric substance, we shall find that air (or some similar gas which closely conforms to Boyle's Law) comes nearest the required standard, and is sometimes used when very accurate measurements

are required. More usually a mercurial thermometer which has been compared with an air thermometer is employed. We shall consider the air thermometer when dealing with the expansion of gases.

19. Sources of error in a mercurial thermometer. The more important sources of error are:—

1. *Change of zero.* In many cases, if an ordinary mercurial thermometer, which has been made for some time, be placed in melting ice, it will be found that its indication is a little higher than the freezing point marked on the scale. The reason of this is referred to in Art. **14**. If the thermometer tube has not been kept long enough before graduation, the gradual contraction of the bulb after graduation causes its interior volume to decrease, and consequently the volume of the contained mercury at 0°, although constant, appears to increase, and the level of the column corresponding to the freezing point rises. To correct this error, it is only necessary to re-determine the freezing point from time to time, and to deduct the observed error from any reading made. Thus, if zero is found, some months after graduation, to be at 0·1° C., and the reading of the thermometer when placed in a bath of water is 64·5° C., the correct temperature of the bath is 64·4° C. (Cf. Exp. **5**.)

2. *Recent heating.* This is a temporary source of error arising from the same cause as *change of zero*. If a thermometer be heated to a high temperature and then allowed to cool, the bulb will for some weeks have a larger volume than the normal. Hence, if the thermometer be placed in a liquid of lower temperature, the mercury will not rise to the same graduation as it normally would, and the temperature it indicates will be lower than the real temperature of the liquid. The same thermometer must therefore not be used for determining low temperatures soon after it has been used for high temperatures. The error is merely temporary, and is, after a day or so, quite inappreciable.

It is for the above reason that, in the construction of thermometers, the freezing point is determined before the boiling point; if the latter were determined first, and the

former soon after, the zero would be marked too low on the scale and would gradually rise, as explained above, under *change of zero.*

3. *Temperature of steam at time of marking boiling point.* The temperature of steam depends on the atmospheric pressure under which boiling takes place. At 760 mm. it corresponds to 100° C. or 212° F.; but if the pressure be greater or less than 760 mm., the boiling point indicates a temperature higher or lower than this temperature.

Tables have been drawn up from experiments by Regnault giving the boiling points corresponding to a wide range of pressures. Hence, if the barometric height be read at the time of determining the boiling point, the temperature corresponding to this height can be obtained from these tables, and the graduation carried out accordingly.

4. *Temperature of the stem.* When a thermometer is used to determine a temperature, it is often impossible to subject more than the bulb and a small portion of the stem to this temperature. The upper portion of the stem is thus at a different temperature from the rest of the tube, and consequently the reading will be slightly different from what it would be if the whole thermometer took up the temperature it is intended to indicate. If the temperature of the bath is higher than that to which the stem is exposed the reading is too low, and if the temperature of the bath is lower than that of the outside temperature the reading is too high. Approximate calculable corrections can be applied in both cases, and will be dealt with in the chapter on the Expansion of Liquids.

5. A thermometer should always be read in the position in which it was graduated; for, if the bore be very fine and the bulb thin, the influence of the pressure due to the contained mercury on the volume of the bulb may cause an appreciable change of the reading when the position of the instrument is altered. It is obvious that the reading will be lowest when the thermometer is held vertical.

6. Theoretically, corrections should be made for the pressure of the air at the time of determining freezing point, and also for the influence of external pressure on

the volume of the bulb; but these sources of error are, under suitable conditions, quite inappreciable.

SOME COMMON AND IMPORTANT TEMPERATURES.

Centigrade.		
7000	...	The sun (approximately)
3500	...	The electric arc
2800	...	Oxy-hydrogen flame
2230	...	Temperature of Bessemer furnace
1710	...	Platinum melts
1500	...	Iron melts Lowest temperature for white heat
1080	...	Copper melts
960	...	Silver melts
400	...	Coal ignites at about this temperature
357	...	Mercury boils
327	...	Lead melts
230	...	Tin melts
115	...	Sulphur melts
100	...	Water boils (= 212° F.)
79	...	Alcohol boils
36·8	...	Temperature of human body in health (= 98·4° F).
17	...	Mean temperature of sea (= 62° F.)
0	...	Water freezes (= 32° F.)
−17·7	..	Mixture of salt and ice (= 0° F.)
−39	...	Mercury freezes
−61·7	...	Record natural lowest temperature
−183	...	Oxygen liquefies
−253	...	Hydrogen liquefies
−268·6	...	Helium liquefies (the lowest temperature yet reached)
−273·1	...	The absolute zero (= − 459° F.)

EXERCISES II.

1. What do we mean by the *temperature* of a substance? State what properties of bodies we employ for the purpose of indicating temperatures.

2. What are the freezing and boiling points of a thermometer? They are commonly called fixed points: how do they differ under ordinary circumstances in respect of being fixed points? A temperature is 20° on Fahrenheit's scale: what is it called on the Centigrade scale?

3. Mercury freezes at $-40°$ C. What is the corresponding temperature on the Fahrenheit scale?

4. To how many Fahrenheit degrees of temperature are 20 Centigrade degrees equal, and what Fahrenheit temperature is the same as 20° C.?

5. Convert the following Centigrade temperatures into their equivalent temperatures in each of the other scales :—

$$850°, \quad -6°, \quad -50°.$$

6. Convert the following Fahrenheit temperatures into their equivalents in each of the other scales :—

$$230°, \quad 27°, \quad -10°, \quad -49°.$$

7. Convert the following Réaumur temperatures into their equivalents in each of the other scales :—

$$110°, \quad -8°, \quad -27°.$$

8. Prove the following formula for the conversion of Fahrenheit temperatures into Centigrade temperatures and *vice versa* :—

$$\tfrac{5}{9}(F + 40) = C + 40$$

$$\text{or } \tfrac{9}{5}(C + 40) = F + 40.$$

CHAPTER III.

APPROXIMATIONS.

20. Every reading of an experimental result obtained in any one of the experimental sciences—Mechanics, Physics, Chemistry, etc.—is only approximate, *i.e.* it is not absolutely true. For example suppose the length of a straight line is measured by an observer with a scale graduated to tenths of an inch. If he is not a trained observer he will give its length as, say, 7·9 inches and a bit more. A trained observer would estimate by eye what fraction of a tenth of an inch the line was over 7·9 inches. Suppose he gives the result as 7·92 inches. We may then assume that the true length of the line is something between 7·915 and 7·925 inches, *i.e.* we can only trust the result to the third significant figure.

Again, suppose in measuring the sides of a rectangle with a scale graduated to millimetres the readings are 4·34 and 2·76 cms., the fractions of a millimetre being estimated by the eye. Since area = length × breadth, one might be inclined to say area = 4·34 × 2·76 = 11·9784 sq. cms. It would, however, be wrong to think that such an answer was correct in its entirety, for the sides have not been measured sufficiently accurately to justify this. The longer side may be anything between 4·335 and 4·345, and the shorter side anything between 2·755 and 2·765. Assuming that the lowest estimates are the correct lengths, the area is found to be 11·9429 sq. cms.; and assuming the largest estimates are correct, to be 12·0139 sq. cms. We have now as the area of our rectangle a minimum estimate 11·9429 and a maximum estimate 12·0139, the difference being ·071 sq. cm., and their mean is 11·9784. The area may thus be mathematically given as 11·9784 $\pm$ ·0355 sq. cms., or in practical

language as 12·0 sq. cms., placing little reliance on the 0. Of course errors made in estimating one dimension may neutralise errors made in estimating the other dimension, but this cannot be depended on.

Again, suppose the above rectangle to be one face of a rectangular box the third side of which measured as before is 1·52 cms. Working out the volume by the usual rule we get $4{\cdot}34 \times 2{\cdot}76 \times 1{\cdot}52 = 18{\cdot}207168$ cub. cms. The volume *may*, however, be as low as $4{\cdot}335 \times 2{\cdot}755 \times 1{\cdot}515$, *i.e.* 18·09353 cub. cms., or as high as $4{\cdot}345 \times 2{\cdot}765 \times 1{\cdot}525$, *i.e.* 18·32123 cub. cms.

To be perfectly honest about it we can only give the volume as 18·2 cub. cms., placing little reliance on the 2; the third figure may be 1 or it may be 3.

It will be noticed that in the above examples the dimensions are measured to three significant figures in each case and the area or volume can only be trusted to three significant figures: hence we get the *general rule* that the result of calculations based upon measurements should only contain as many significant figures as there are in the least accurately measured datum.

Note.—Ciphers do not count as significant figures unless they lie between numbers which are not ciphers. Thus in either of the numbers 3002500 or ·030025 there are five significant figures, viz. 30025.

In consequence of the rule above approximate methods of calculation* can be used in nearly all physical work, with a great saving of time and trouble, and without sacrificing the accuracy of the work. If the number of factors to be multiplied or divided is large 4-figure logarithms are a great boon. These logarithms are quite accurate enough for most physical calculations. It is a waste of time to use 7-figure logarithms.

21. Small quantities occur frequently in problems on expansion, *e.g.* small increments of length, and the arithmetic containing these quantities is apt to consume much time unless some artifices are employed.

* See *Tutorial Arithmetic*, Chap. xviii.

(*a*) Note that the square or cube (or any higher power) of any very *small* quantity is very small in comparison with that small quantity itself. Thus $(0{\cdot}002)^2 = 0{\cdot}000004$.

$$(0{\cdot}002)^3 = 0{\cdot}000000008.$$

Also the product of any two small quantities is negligibly small in comparison with either of those quantities. Thus

$$0{\cdot}002 \times 0{\cdot}003 = 0{\cdot}000006.$$

It is evident then that in such an expression as

$$1{\cdot}031 + 0{\cdot}002 + (0{\cdot}002)^2,$$

where the quantities are all derived from measurement, the correct answer is not 1·033004, but simply 1·033, for the third quantity is of much less importance than the possible errors in the first two quantities.

Example. What is the area of a square whose side when measured is found to be 1·003 cms. ?

By direct multiplication $(1{\cdot}003)^2 = 1{\cdot}006009$. The datum is accurate to four significant figures, hence this is also the accuracy of the answer, hence the area = 1·006 sq. cms. Using an elementary theorem in Algebra, viz.

$$(a + b)^2 = a^2 + 2ab + b^2,$$

we see that

$$\begin{aligned}(1 + 0{\cdot}003)^2 &= 1 + 2 \times 0{\cdot}003 + (0{\cdot}003)^2 \\ &= 1 + 2 \times 0{\cdot}003 \text{ by the above} \\ &= 1{\cdot}006.\end{aligned}$$

Hence, as a general rule, we may say :—When a is small a^2 is negligible and

$$(1 + a)^2 = 1 + 2a.$$

Similarly $(1 + a)^3 = 1 + 3a + 3a^2 + a^3$ and in practice is equal to $1 + 3a$.

Again $(1 + a)(1 + \beta) = 1 + a + \beta + a\beta$. This when both a and β are negligibly small $= 1 + a + \beta$: for, in comparison with a and β, $a\beta$ is negligible.

Example. The sides of a rectangle are measured and found to be 1·00062 feet and 1·00045 feet. Find the area.

$$\begin{aligned}\text{Area} &= (1{\cdot}00062)(1{\cdot}00045) \\ &= 1 + {\cdot}00062 + {\cdot}00045 \\ &= 1{\cdot}00107.\end{aligned}$$

It may, however, be anything between 1·00106 and 1·00108.

(*b*) Again, by actual division, the reciprocal of $1 - a$ where a is less than 1 is given by

$$\frac{1}{1 - a} = 1 + a + a^2 + a^3 \ldots$$

$\therefore$ When a is very small $\dfrac{1}{1 - a} = 1 + a$.

Similarly under the same conditions

$$\frac{1}{1+\alpha}, \text{ which} = 1 - \alpha + \alpha^2 - \alpha^3 \ldots .$$

reduces to $1 - \alpha$.

Also if both α and β are small

$$\frac{1+\alpha}{1+\beta} = (1+\alpha)\left(\frac{1}{1+\beta}\right) = (1+\alpha)(1-\beta).$$
$$= 1 + \alpha - \beta.$$

Example. Divide 1·00038 by 1·00097.

$$\frac{1{\cdot}00038}{1{\cdot}00097} = 1 + 00038 - {\cdot}00097 = 1 - {\cdot}00059$$
$$= {\cdot}99941.$$

(*c*) **A summary** of the above and simple results is here given. Remember that for the results to be true α and β must be very small in comparison with unity.

$$(1 \pm \alpha)(1 \pm \beta) = 1 \pm \alpha \pm \beta$$
$$(1 \pm \alpha)^2 = 1 \pm 2\alpha$$
$$(1 \pm \alpha)^3 = 1 \pm 3\alpha$$
$$\frac{1}{1+\alpha} = 1 - \alpha \text{ and } \frac{1}{1-\alpha} = 1 + \alpha$$
$$\frac{1 \pm \alpha}{1 \pm \beta} = 1 \pm \alpha \mp \beta$$

(*d*) **In working out numerical problems** students will sometimes obtain recurring decimals. Now, on thinking of the real meaning of a recurring decimal it is evident that such a thing can have no place in physics. The nearest approximation must be given in all cases.

Example. The perimeter of an equilateral triangle is found by measurement to be 25·1 cms. Find the length of the side.

Side = $\frac{1}{3}$ of circumference = $\frac{1}{3}$ of 25·1 = 8·3$\dot{6}$.

This, however, means 8·36666666 . . . . *i.e.* you have assumed an infinite accuracy, which is, of course, quite unjustifiable. The result should be stated as 8·37, the 7 even being very doubtful.

Again, working on another method, the length of the perimeter may be anything between 25·05 and 25·15, *i.e.* the length of one side may be anything between 8·35 and 8·38, ∴ the answer given above, viz. 8·37, is not far wrong.

(*e*) **It is also easy** to prove that squaring an inaccurate number doubles the inaccuracy, and cubing it trebles the inaccuracy.

It also follows that if the square root of a slightly inaccurate number be taken the inaccuracy is halved.

Throughout this book students should carefully observe how the above rules are applied. Contracted methods of

working should be used whenever possible. "When calculating from observed quantities, it is dishonest to use more figures than we are sure of." (Prof. J. Perry in *Practical Mathematics*.)

EXERCISES III.

1. The numbers 1·312, 9·563, 3·569, 4·213 are known correct to 3 decimal places. Determine their sum and indicate the limits of error.

2. Determine the limits of error in $4·213 - 3·569$ (correct to 3 places).

3. The numbers 1·7324 and 2·5968 are correct to 4 places of decimals only (*i.e.* correct to 5 significant figures). How far can their product be relied on?

4. A line is measured and found to be 4·32 inches long. Express its length in centimetres given that 1 cm. = ·3937 in.

5. The radius of a circle is found to be 2·46 cms. Find the area.

6. Two thermometers, one (A) with a Centigrade graduation, the other (B) with a Fahrenheit graduation, when immersed in the same liquid, both gave respectively the following readings:—

A.	B.
− 10·1	14·5
0	32·4
10·2	50·6
19·8	68·3

Calculate a table to show the correction to be applied to the Fahrenheit thermometer at each temperature, assuming the Centigrade thermometer to be correct.

By plotting a curve deduce also the probable correction to be applied when B reads 20°, 30°, 40°, 60°.

Note.—The thermometers are only read to the first decimal place.

Find the approximate value of—

7. $(1·000024)\,(1·000065)$. [*Type* $(1+\alpha)(1+\beta)$].

8. $(1·00018)\,(·99982)$. [*Type* $(1+\alpha)(1-\beta)$].

9. $(1·00035)^2$; $(·999987)^3$. [*Type* $(1+\alpha)^2, (1-\alpha)^3$].

10. $\frac{1}{1·000025}$; $\frac{1}{0·9987}$. [*Type* $\left(\frac{1}{1+\alpha}\right)$, $\left(\frac{1}{1-\alpha}\right)$].

11. $\frac{1·00016}{1·00004}$; $\frac{1·00018}{0·99986}$. [*Type* $\frac{1+\alpha}{1+\beta}$; $\frac{1+\alpha}{1-\beta}$].

12. $100\left[\frac{1+10(·000064)}{1-25(·000008)}\right]$; $\frac{100}{1·00016}$. [*Type* $n\frac{1+\alpha}{1-\alpha}$; $\frac{n}{1+\alpha}$].

CHAPTER IV.

EXPANSION OF SOLIDS.

22. In Art. **7** it was shown that when a solid rod is heated its length increases. The increase of length is, however, very minute, and delicate means have to be devised for measuring it. Before proceeding to quantitative measurements it is well, however, to examine whether all substances expand on heating, and if so whether they all have the same expansibility.

Exp. 7. Take two similar strips of steel and copper each about 10 ins. long, $\frac{1}{2}$ in. wide, and $\frac{1}{32}$ in. thick, and solder or rivet them together along their whole length so that they are straight when cold. Heat the compound strip in the flame. Note what happens. The strip becomes curved with the copper on the convex side, *i.e.* the longer side (Fig. 15). This shows that copper expands more than steel.

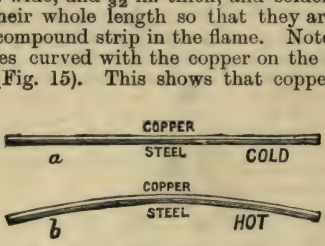


Fig. 15.

Exp. 8. Fuse a piece of copper wire into a glass rod. What happens when the rod cools? The glass fractures, the reason being that the copper contracts more than the glass, thus pulling the glass to which it is stuck away from the rest.

Exp. 9. Repeat the last experiment, substituting platinum for copper. No fracture occurs on cooling, showing that platinum and glass contract (and therefore expand) at about the same rate.

The result of experiments such as the above is to show that all common substances expand on heating * and contract on cooling, the rates of expansion being different for different substances.

23. Linear and cubical expansion. When a solid is heated it in general expands, and unless the substance is non-isotropic,† such as a crystal, it expands equally in all directions. For example, if a cube of an isotropic substance is heated each edge increases equally in length, each face increases equally in area, and the volume of the cube, as a whole, also increases. Increase of length is spoken of as *linear expansion*, increase in area as *superficial expansion*, and increase in volume as *cubical expansion*. The study of the amount of expansion may be made by exact measurements of change of length or by exact measurements of change of volume. When length measurements are made linear expansion is directly determined, and the corresponding superficial and cubical expansions are deduced from the result. When volume measurements are made cubical expansion is determined directly, and the corresponding linear and superficial expansions are deduced from the result. Hence in the measurement of expansion the determination of linear expansion or the determination of cubical expansion may be made the starting point.

24. Rate of expansion with temperature. It must be remembered that the thermometric scale of temperature is derived from the expansion of mercury in a glass tube, and that successive degrees of temperature correspond to

* A rod of stretched indiarubber contracts on being heated, but this is not a true contraction. Hot indiarubber is not so elastic as cold indiarubber, hence the stretching force, when the rubber is hot, is not able to maintain the elongation which it did when the rubber was cold.

† An isotropic body is a body whose properties are the same in all directions. Examples are metals, liquids, gases. The treatment in the text will refer only to isotropic bodies.

successive equal * increments of volume. Hence if we measured the expansion of any body degree by degree and found the same expansion for each degree rise of temperature we might say that the rate of expansion with temperature was uniform, but such a statement would merely mean that the body expands in exactly the same way as mercury in glass appears to expand.

It will be found that as a general rule substances expand in approximately the same way as mercury, that is, the increment of length or volume for each degree is approximately the same, and the expansion for one degree rise of temperature is approximately $1/n^{\text{th}}$ of the expansion for n degrees rise.

25. Coefficient of linear expansion. The following simple law has been established by experiments:—

If different bodies of the same material are heated through the same range of temperature their lengths increase *by the same fraction of their lengths at* 0° C.

For example, if the temperature of any body made of copper is raised 1° C. on any part of the scale, then the increase of its length is found to be $\frac{1}{60000}$ of its length at 0° C. The fraction $\frac{1}{60000}$ is called the **coefficient of the linear expansion** of copper.

DEFINITION.—**The coefficient of linear expansion of the substance composing a body is the ratio**

$$\frac{\text{increase in length for 1° C. rise in temperature}}{\text{length at 0° C.}}.$$

* It should, however, be noticed that the increments of volume for successive degrees of temperature are not strictly equal. In constructing a thermometer the volume of the bore between the fixed points is divided into equal divisions, but in using the instrument the volume of each division increases with the expansion of the glass tube as the temperature rises, so that the increment of volume for each successive degree slightly increases, and the increment of volume from, say, 99° C. to 100° C. is therefore greater than that from 0° C. to 1° C. by the increase in volume of a degree division due to the expansion of the glass tube between 1° C. and 100° C.

Simple problems on linear expansion are therefore easy applications of the Rule of Three. For harder problems it is preferable to employ a formula.

26. Formula for linear expansion. Let the length of a body at 0° C. be L_0 and at $t°$ C. be L_t, and let l be the coefficient of linear expansion.

Then increase of length when heated $t°$ C. $= L_t - L_0$;

$\therefore$ " " " $1°$ C. $= (L_t - L_0) \div t$.

Thus coefficient of linear expansion $= \dfrac{(L_t - L_0) \div t}{L_0}$;

$$i.e. \quad l = \frac{L_t - L_0}{L_0 t}.$$

Obviously this formula may also be written in the form

$$L_t = L_0 (1 + lt).$$

27. Surface expansion. A body has the same coefficient of expansion whether we consider the expansion in length, breadth, or thickness. Knowing the increase in length and breadth, we can find by ordinary mensuration the increase in area or surface.

DEFINITION.—**The coefficient of surface expansion of a body is the ratio**

$$\frac{\textit{increase in surface for } 1° \textit{ C. rise in temperature}}{\textit{surface at } 0° \textit{ C.}}.$$

Thus, if S_0 = area of body at 0° C.,

S_t = " " $t°$ C.,

s = coefficient of surface expansion,

then, arguing as in Art. 26, we obtain

$$s = \frac{S_t - S_0}{S_0 \cdot t},$$

or $$S_t = S_0 (1 + st).$$

Next consider any portion of the surface in the shape of a square* whose side measures L_0 at 0° C. and L_t at $t°$ C.

---

* In the case of a curved surface, *e.g.* a sphere, only *very small* squares can be drawn upon it.

Then increase in area for $t°$ C. heating $= L^2_t - L^2_0$

$$= L^2_0 (1 + lt)^2 - L^2_0$$

$$= L^2_0 (2lt + l^2 t^2);$$

$\therefore$ increase in area for 1° C. heating $= L^2_0 (2l + l^2 t)$;

$\therefore$ coeff. of surface expansion for this square $= \dfrac{L^2_0 (2l + l^2 t)}{L^2_0}$

$$= 2l + l^2 t.$$

But for any solid l is very small, and therefore l^2 is negligible.

Hence the coefficient of surface expansion for this square $= 2l$.

This coefficient determines the rate of expansion for any square of any size which can be drawn anywhere on the surface. Hence the rate of expansion for the whole surface must be the same. Thus

coeff. of surface expansion = 2 × coeff. of linear expansion.

Thus, to take an example, the increase in the surface of a copper body per rise of temperature of 1° C. is $\left(2 \times \dfrac{1}{60000}\right)$ or $\dfrac{1}{30000}$ of its surface at 0° C., *i.e.* **the coefficient of surface expansion** of copper is $\dfrac{1}{30000}$.

28. Volume expansion.—

DEFINITION.—**The coefficient of volume expansion (or cubical expansion) is the ratio**

$$\frac{\textit{increase of volume for } 1° \textit{ C. rise in temperature}}{\textit{volume at } 0° \textit{ C.}}.$$

Thus, if V_0, V_t be the volumes at 0° C. and $t°$ C., and if c be the coefficient of cubical expansion, then

$$c = \frac{V_t - V_0}{V_0 t} \text{ and } V_t = V_0 (1 + ct).$$

Next consider any portion of the volume in the shape of a cube, whose side measures L_0 at 0° C. and L_t at $t°$ C. Then, arguing as in Art. 27, it will be found that

coeff. of volume expansion for this cube

$$= 3l + 3l^2 t + l^3 t^2 = 3l,$$

since l^2 and l^3 are negligible.

This coefficient determines the rate of expansion of any cube of any size taken anywhere within the volume of the body. Hence the rate of expansion for the whole volume must be the same. Thus

coeff. of volume expansion = 3 × coeff. of linear expansion.

Thus, to take an example, the increase in volume of a copper body per rise of temperature of 1°C. is $\left(3 \times \frac{1}{60000}\right)$ or $\frac{1}{20000}$ of its volume at 0° C., *i.e.* the **coefficient of volume expansion** of copper is $\frac{1}{20000}$.

The last two theorems also admit of an easy geometrical proof.

We shall see later that the volume expansion, and consequently the linear expansion, of a solid substance may be obtained indirectly by observing the expansion of a liquid in a vessel composed of that substance.

29. Lengths and volumes at different temperatures. Let L_0, L_t, $L_{t'}$ be the lengths of a rod at 0° C., t° C., and t'° C., t' being greater than t, and let l be the coefficient of linear expansion of the substance composing the rod.

Then
$$L_t = L_0 (1 + l t)$$
$$L_{t'} = L_0 (1 + l t')$$
$$\therefore \frac{L_{t'}}{L_t} = \frac{1 + l t'}{1 + l t} = (1 + l t')(1 - l t) = 1 + l (t' - t) \text{ approx.}$$
$$\therefore L_{t'} = L_t (1 + l (t' - t)) \quad \ldots\ldots(1)$$

a very useful relation when the length is given at some temperature other than 0° C.

The student should prove in like manner that

$$V_{t'} = V_t (1 + c (t' - t)) \quad \ldots\ldots(2)$$

Formulae (1) and (2) are only approximate and should not be used when the coefficients of expansion and the difference of temperature are large. Scarcely any error is obtained when they are applied to solids. In the case of liquids their application is sufficiently accurate for most purposes, but in the case of gases the expansion is far too great to admit of these approximations.

Examples. In problems on expansion it is better to work from first principles than to calculate mechanically from the formula.

(1) *Having given the length of a solid at one temperature and its coefficient of expansion, to find the length at another temperature.*

The length of a copper rod at 10° is 200·34 ins. Find its length at 100° C., the mean coefficient of expansion being ·000017.

Difference of temperatures is $100 - 10 = 90°$. A length 1 at 10 increases by $90 \times \cdot000017$ or $\cdot00153$ when heated to 100°.

Since 1 increases by $\cdot00153$,

$\therefore$ 200·34 ins. increase by $\cdot306$,

i.e. the length at 100° C. is 200·65 ins.

Note.—In working this example it is assumed that the rod expands $\cdot000017$ of its length at 10° C. for each degree of increase of temperature. This is not strictly true. The rod expands $\cdot000017$ of its length at 0° C. ; but, as its length at 10° is very nearly the same as at 0°, no appreciable error is made.

(2) *To find the coefficient of expansion, having given the length, volume, or area at two different temperatures.*

The volume of a piece of glass at 100° C. is 100·258 c.c., and its volume at 0° is exactly 100 c.c. Find the mean coefficient of cubical expansion.

$$100\cdot258 \text{ c.c.} - 100 \text{ c.c.} = \cdot258 \text{ c.c.}$$

The increase in volume for 100° C. is $\cdot258$ c.cs. Hence for 1° it is $\cdot258/100$ or $\cdot00258$.

Hence the coeff. of cubical expansion is

$$\frac{\cdot00258 \text{ c.c.}}{100 \text{ c.c.}} \text{ or } \cdot0000258.$$

The coefficient of linear expansion is $\frac{1}{3}$ of this, or $\cdot0000086$, and that of surface expansion is double this, *i.e.*

$$2 \times \cdot0000086 \text{ or } \cdot0000172.$$

(3) The coeffieient of linear expansion of glass is 0·0000083. Find the volume at 15° C. of a glass flask of exactly 1 litre capacity at 0° C.

In dealing with hollow vessels note that if the space inside was exactly filled up with the same medium as the vessel is composed of, and the vessel and contents were heated, the matter put in would exactly fill the vessel at all temperatures. But the volume of the matter put in is equal to the capacity of the vessel, hence the increase of volume of a hollow vessel depends upon the volume expansion of the matter composing the vessel.

The coefficient of volume expansion of glass is 0·000025. The increase in capacity of the flask per degree is $0\cdot000025 \times 1000$ and therefore the increase for 15° C.

$$= 0\cdot000025 \times 1000 \times 15 \text{ c.cs.}$$

$$= 0\cdot37 \text{ c.cs.}$$

Hence capacity at 15° C. = 1000·37 c.cs.

30. Laboratory methods of measuring the coefficient of linear expansion. The expansion of most solid bodies with rise of temperature is so small that delicate means of measurement have to be adopted. In the method usually adopted either a microscope or a micrometer screw is used.

Microscopes suitable for the purpose must be of low power, *i.e.* the field of view should be about 2 mm. diameter. A convenient form is that provided with an eyepiece scale, the value of whose divisions in millimetres is easily obtained by focussing the microscope upon a scale graduated to tenths of a millimetre and noting coincidences at both ends of the field. For instance, suppose 32 eyepiece-scale divisions exactly cover 8 tenths of a millimetre, then 1 eyepiece-scale division represents 0·0025 cm.

Another form of microscope is provided only with cross wires, but it slides along a slotted base and its position is measured by a very accurate vernier.*

Micrometer screws suitable for the purpose may be of either the screw-gauge or spherometer type.* The essential principle in each is an accurately made screw of fine pitch, generally $\frac{1}{2}$ mm., working in a fixed collar. Graduations on the collar indicate the number of complete turns the screw has progressed, while graduations on a disc fixed perpendicularly to the end of the screw indicate the fraction of a complete turn.

Exp. 10. To determine the coefficient of linear expansion of brass. Take a brass tube not less than half a metre in length and about 5 mm. in internal diameter. Near one end file a deep notch, N, and near the other file a sharp clear line, A. Measure with a millimetre scale the distance between A and N. Mount the tube horizontally, the notch fitting over a knife-edge on which it can be clamped, and the other end resting on the stage of a microscope so that A can be viewed. Work on a large stone bench if possible. Attach rubber tubes to the ends of the brass tube. Pass a current of cold water through the tubes from the supply system. Take the reading of A on the eyepiece scale. Take the temperature of the water as it leaves the tube. Now pass a current of steam through. Observe—in the microscope—the gradual motion of A outwards from N. When A has come to rest read its position again. At the

* Full details of these instruments should be obtained either from the instruments themselves or from a book of Practical Physics. (See Bower and Satterly's *Practical Physics*, Chap. iii.)

same time read the temperature of the steam in the boiler. (This is hardly necessary : if the barometer is near the normal the steam temperature may be assumed to be 100° C.)

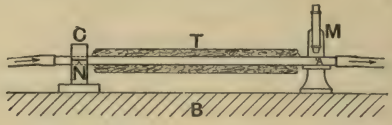


Fig. 16.

To prevent condensation in the tube and the possibility of its temperature not being that of the steam, the tube may be wrapped around with a layer, T, of cotton wool. To make sure that the readings are correct, pass cold water again through the tube and see if A returns to its old position.

From the two microscope readings find the distance in millimetres through which A has moved. Divide this by the product of the length, N A, and the difference in temperatures of the cold water and the steam : the result is the coefficient of expansion of brass.

Example. In a certain experiment, when water of temperature of 10° C. was passing through the tube the reading of A was 2·4, when steam at 100° C. was passing the reading was 43·6. The length of N A was 60·0 cms. and 1 division of the eyepiece scale was equal to 0·0025 cm.

Then the coefficient of expansion $= \dfrac{(43{\cdot}6 - 2{\cdot}4) \times {\cdot}0025}{60{\cdot}0 \times 90} = {\cdot}000019.$

If the brass is in the form of a rod it may be mounted on corks, C, C,* and placed inside a glass tube (as in Fig. 17) through

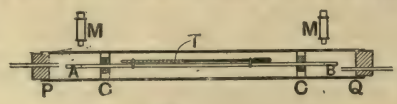


Fig. 17.

which the water and steam can be passed. In this case it is better to use two microscopes, M, M, focussed on two fine lines, A and B, one on each end of the tube. A thermometer may be tied on to the rod itself. Readings at an intermediate temperature, say 50° or 60°, may be obtained by slowly passing warm water from a large tank through the tube. Put a thermometer in the tank and in the

* Holes must be bored through the corks to allow free circulation of water and steam.

outlet water, and make quite sure that the temperature and the positions of A and B are stationary before reading them. Show from the results obtained that the expansion is proportional to the change of temperature.

Exp. 11. Find the coefficient of expansion of iron. As before, take an iron tube, close up one end by a flat piece of metal, and solder or screw into the tube near it a short tube to act as an outlet. Mount as before, but instead of a microscope take a micrometer screw,* and, by fixing the collar in a clamp, arrange the clamp so that the screw itself can be brought up to the flat cap. Take readings with cold water and steam as before.

31. Historical methods of measuring the coefficient of linear expansion. The most important of these are the indirect method due to Laplace and Lavoisier, and the direct method of Roy and Ramsden.

Laplace and Lavoisier magnified the increase in length by mechanical and optical means. Their results are not very accurate.

32. Roy and Ramsden measured *directly* the expansion of the rod by a micrometer screw. The pitch of the screw was $\frac{1}{50}$ inch and the head was divided into 100 parts, so that measurements could be made as fine as $\frac{1}{5000}$ in. The method, as slightly modified by later experimenters, is shown *diagrammatically* in Fig. 18. Three troughs are placed on a table parallel to one another on props. In each of the two outer troughs is placed a metal rod, *A B*, *E F* respectively, and on props in the middle trough is placed the rod *C D*, whose coefficient of expansion is to be measured. The three rods are approximately of the same length and near each extremity is fixed vertically a small upright. The uprights on *A B* carry cross wires *a* and *b*, as shown in Fig. 18 (*a*). The uprights on *E F* carry horizontal rods on which are fixed cross wires *e* and *g* and equal lenses

* A spherometer is useful for this experiment. If one of the legs is fixed in the clamp, the screw can be brought to touch the flat piece, closing the end of the tube. If a screw gauge is used, the flat piece closing the end of the tube must project sideways, so that the butt of the gauge does not come in the way. Unless these instruments can be studied at first hand, read up their descriptions in Bower and Satterly's *Practical Physics*.

f and h. The upright at the C end of $C\ D$ carries a lens c, the upright at the D end carries a frame in which slides

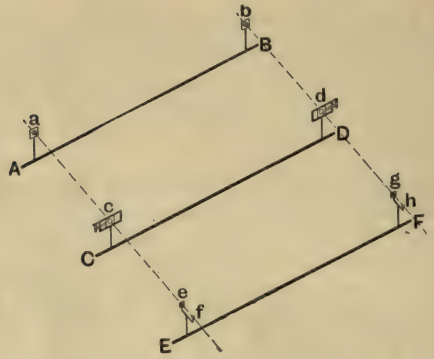


Fig. 18.

another frame (see Fig. 18 (b)) carrying a lens d exactly equal to c. The lens frame can be moved parallel to the length of the bar by means of an attached micrometer screw, s. The system $a\ c\ e\ f$ forms a telescope in which an image of a formed by c is viewed through f, and its position relative to e observed. The system $b\ d\ g\ h$ forms another exactly similar telescope. The three troughs are filled with melting ice and the rods adjusted until on placing the eye at f the intersection of the cross wires e exactly covers the image of the intersection of the cross wires a. Placing the eye at f, the micrometer screw is adjusted until the intersection of cross wires g exactly covers the image of the intersection of cross wires b. The reading of the screw is now taken.

Fig. 18 (a).

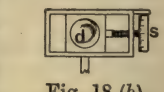


Fig. 18 (b).

The ice in the central trough is now melted and the water heated to boiling. $C\ D$ expands and most likely c moves to the left of the straight line $a\ e\ f$ and d to the

right of the straight line *b g h*. The centre trough is now gently moved until *c* is brought back into line with *a e f*, and now the micrometer screw is again adjusted until *d* is brought into line with *b g h*. The reading is again taken and the difference in the two readings gives the amount lens *d* has been moved to the left, *i.e.* the amount the length of the rod *C D* between the uprights has expanded on being heated from 0° C. to 100° C. From this the coefficient of linear expansion was deduced.

This method is interesting, as it was devised by Ramsden and Roy to determine the true lengths of the metal rods used in measuring the base-line on Hounslow Heath when making the first Ordnance Survey of Great Britain.

TABLE OF COEFFICIENTS OF LINEAR EXPANSION.

These are *mean coefficients between* 0° C. *and* 100° C. As they differ slightly for different specimens of the same substance it must be understood that they are only approximate.

Invar (a nickel steel)	·0000009
Quartz	·0000017
Glass	·0000086
Platinum	·0000089
Cast Iron	·000011
Steel	·000012
Copper	·000017
Brass	·000019
Silver	·000019
Lead	·000028
Tin	·000023
Ice	·00004
Oak	·00005
Sulphur	·00006

The coefficients above are given with reference to the Centigrade scale, but they may be converted to refer to any other scale. The student, for example, should prove that the coefficient of expansion per degree Fahrenheit is equal to five-ninths of the coefficient of expansion per degree Centigrade.

33. Cubical expansion of solids. It will be understood after a study of the Expansion of Liquids that the measurement of the cubical expansion of solids involves exactly the same methods as those adopted for the measurement of the cubical expansion of liquids, and that the experimental study of the cubical expansion of solids and liquids is practically one subject.

In the early stages of research into the expansion of solids and liquids all experimental work on the expansion of liquids was complicated by the expansion of the vessel containing it, and this presented a serious obstacle to the advance of research. The results obtained always involved the coefficients of expansion of a liquid and a solid, and neither could be accurately determined unless the other was known. If the absolute expansion of any one liquid or solid were known this could be taken as a starting point for the determination of other coefficients. Attempts were made to obtain a starting point of this kind by making exact determinations of linear expansion (Art. 31), but at that time it was found impossible to make determinations of sufficient accuracy. This difficulty was, however, done away with by the discovery of Dulong and Petit's method of determining the absolute expansion of a liquid, and Regnault's determination of the coefficient of absolute cubical expansion of mercury gave the starting point of all accurate determinations of coefficients of cubical expansion for solids and liquids.

34. Practical precautions necessary on account of expansion of metals by heat. The expansion of metals is very frequently a source of trouble. Thus, in constructing a railway, small spaces have to be left between the metals to allow for expansion due to the extreme yearly range of temperature. Similarly, allowance must be made for expansion in constructing an iron bridge, and it is often difficult to secure rigidity and yet allow each part freedom of expansion. The tubular girders of the Britannia Bridge are mounted on rollers at each end, so that they may be free to expand, without straining the structure. For the same reason, the joints of water pipes and gas mains

are made like those of a telescope. If furnace bars are firmly fixed they become bent, owing to the expansion which they undergo on heating; for this reason the ends of the bars should be free to move in the direction of their length. Metal castings often break in cooling, on account of one part cooling more rapidly than another, and thus producing great strains in the metal. For example, in casting a wheel with a thin rim but massive arms, the rim solidifies and cools much more quickly than the arms, and as a consequence the contraction of the latter is opposed by the rim, and a stress is set up which may fracture the rim or one or more of the arms. To avoid this, the arms are often bent, so that the strain thus produced is relieved as they straighten. In all measurements made by metal rods or chains the temperature must be noted, and a correction applied. The imperial standard yard is the distance, at 62° F., between two fine lines engraved on gold plugs, which are inserted into a bronze bar kept in the Standards Office.

In the construction of clocks, chronometers, and watches of the best kind, great precaution has to be taken to obviate the effects of expansion on the time-keeping part of the mechanism. The rate of a clock is generally controlled by a pendulum, whose time of vibration varies as the square root of its length, or, more accurately, as the square root of the distance between two points called respectively "the centre of suspension" and "the centre of oscillation."* If the temperature varies this distance varies, and the clock goes "slow" as the temperature rises and "fast" as the temperature falls. Compensating pendulums are constructed to prevent this change of rate with change of temperature.

35. Harrison's gridiron pendulum (Fig. 19) is the best known form. It consists of four oblong frames arranged as shown in figure. The vertical rods are alternately of steel (S) and brass (B). The central steel rod passes through holes in the lower horizontal pieces of the frames,

* See *First Stage Mechanics of Solids*, § 239; or *Tutorial Dynamics*, § 287.

and carries the bob at its lower end. The arrangement is such that the steel rods expand downwards, while the brass rods expand upwards; and if the downward and upward expansions be equal, the centre of the bob is neither raised nor lowered. If it were possible to have the total length of each set of rods the same, only one metal would be required, but the inner rods must necessarily be shorter than the outer ones, and therefore, if compensation is to be exact, they must be made of the more expansive metal. The condition of compensation therefore requires that the total expansion of one set of rods should be equal to the total expansion of the other, the rods being so arranged that these expansions take place in opposite directions. Let L_0 denote the total length at 0° C. of one set of rods, and L'_0 the total length of the other set; then—

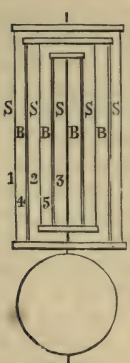


Fig. 19.

$$L_t = L_0 (1 + \alpha t), \text{ and } L'_t = L'_0 (1 + \beta t),$$

where α and β are the corresponding coefficients of linear expansion. Hence, in one case, the expansion is given by—

$$L_t - L_0 = L_0 \alpha t,$$

and in the other by—

$$L'_t - L'_0 = L'_0 \beta t.$$

For compensation we must have—

$$L_0 \alpha t = L'_0 \beta t, \text{ or } L_0 \alpha = L'_0 \beta, \text{ or } \frac{L_0}{L'_0} = \frac{\beta}{\alpha}.$$

That is, the total effective lengths* of the rods in each set must be taken inversely proportional to the mean coefficient of expansion of the metals of which they are made. For steel and brass the ratio $\frac{\beta}{\alpha} = \frac{2}{3}$ nearly.

* It must be here noticed that the two vertical bars of each frame only effect the same expansion of the frame as one bar of the same length. Hence, in the figure, the effective length of the bars is (1 + 2 + 3) for steel and (4 + 5) for brass.

Graham's mercurial pendulum (Fig. 20). The bob of this pendulum consists of a frame carrying two glass cylinders containing mercury. The principle of compensation is the same as that described above; the rod to which the bob is attached expands downwards, while the mercury expands upwards, and, if the quantity is properly adjusted, the final result is that the distance between the centres of oscillation and suspension is unchanged, and therefore the rate of the clock is unaffected by changes of temperature.

Fig. 20.

36. In chronometers and watches the rate is controlled by balance wheels, which oscillate under the influence of a steel hair-spring. The time of oscillation increases if the dimensions are increased, and therefore a rise of temperature causes the rate of the watch to diminish. Further, a rise of temperature lessens the stiffness of the hair-spring, and this also tends to increase the time of oscillation, and diminish the rate of the watch. In chronometers and the best made watches, compensation is effected by making the circumference of the wheel of two metals, the outer being the more expansible. This double rim is made in three parts, each supported by an arm of the wheel (Fig. 21). The effect of rise of temperature on this arrangement is, that while the end of segment of the rim nearest the supporting arm is pushed out from the centre by the expansion of that arm, the other end curls in towards the centre of

Fig. 21.

the ring owing to the greater expansion of the outer strip of metal. By properly adjusting the screw weights, carried by each segment, exact compensation can thus be obtained.

37. Invar. M. Guillaume, a French metallurgist, has recently invented an alloy of nickel and steel containing 36% of nickel, whose coefficient of expansion is very nearly zero, whence its name Invar. It is very likely that the use of Invar will render obsolete all methods of compensating pendulums; Invar will also be employed for standards of length and capacity, and in all cases where the expansion of a metal by rise of temperature introduces complications.

38. Force of expansion and its applications. The force exerted by a metal rod, if prevented from expanding or

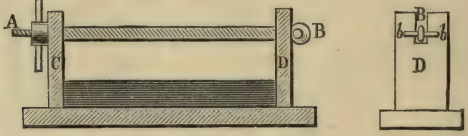


Fig. 22.

contracting as its temperature changes, is very great. For example, if an iron bar, of one square inch cross-section, is heated from 0° C. to 100° C., and then fixed, so as to prevent contraction, it will exert a force equal to the weight of about thirteen tons on the fixings. This may be illustrated by the simple apparatus shown in Fig. 22. The bar, A B, is held in position in a cast-iron frame, C D, by a short cast-iron rod, *b*, passing through the holes at the end B, outside the upright D. The bar is then heated by a lamp and the screw at the end A is tightened. The bar is then allowed to cool, and, up to a certain point, contraction is resisted by the cast-iron rod; finally, however, the stress becomes too great, and the rod is broken in two. This contractile force of metals, due to fall of temperature, has many useful applications. The tyres of ordinary carriage and cart wheels are fitted on when red hot, and, on cooling, bind the

wheels firmly together. Boiler plates are riveted with red-hot rivets, which, on cooling, draw the plates so close together that a steam-proof joint is formed. In cases where the walls of buildings have bulged outwards, they have been drawn in by passing iron bars through them, across the building, and attaching to their extremities, outside the building, iron plates screwed up to the wall with a nut. The bars are then heated, the nut and plate screwed up tightly against the wall, and the bar allowed to contract, drawing the walls with it. This process is repeated, until the walls are made to assume their proper positions.

The expansion of a body due to heat may be sometimes turned to good account when a stopper of a glass bottle gets tight. Hold the neck of the bottle as close to a flame as possible. The neck gets hot before the stopper and due to the expansion of the neck the stopper becomes loose. If the neck is held in the flame too long the stopper also becomes heated and things are as bad as before.

39. Metallic thermometers. Thermometers, based on expansion, have been constructed with metals. They are of little use as accurate instruments, but are convenient for giving an automatic record of the variation of temperature, and thus act as maximum and minimum thermometers.

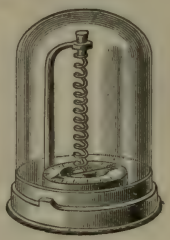


Fig. 23.

The best known instrument of this class is **Breguet's thermometer** (Fig. 23). If two strips of metal of different expansibility be riveted or soldered together and wound into a spiral with the most expansible metal inside, then any rise of temperature causes the inner strip to expand more than the outer, and consequently the spiral unwinds. Similarly, if the temperature falls, the spiral coils up. This thermometer is an application of this fact. It consists of a spiral composed of three metallic strips of silver, gold, and platinum, soldered together so as to form a single ribbon. The silver, which is the most expansible, is placed inside,

the platinum outside, and the gold between serves to connect them. The spiral carries, at its lower end, a horizontal needle, which traverses a dial. As the spiral winds or unwinds with change of temperature, the needle is deflected in a corresponding direction, and indicates, by its motion, the variation of temperature.

40. Change of density with temperature. We have seen that, in general, when a body is heated it expands—that is, its volume increases—and, since the mass of the body remains constant, it must necessarily follow that its density decreases. For, if V_0 denote the volume at 0° C. and V_t the volume at $t°$, also if d_0 denote the density at 0° C. and d_t the density at $t°$ C., then, since

$$\text{Mass} = \text{Volume} \times \text{Density}^*$$

and the mass of the body remains constant—

$$V_0 d_0 = V_t d_t.$$

That is—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0}.$$

But we know that $V_t = V_0 (1 + ct)$, where c denotes the coefficient of volume expansion.

Therefore—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0} = \frac{V_0 (1 + ct)}{V_0} = 1 + ct,$$

or—

$$d_t = \frac{d_0}{1 + ct}.$$

This is true, as it stands, for solids, liquids, and gases; but for solids and some liquids we may have, *when t is small enough*, an approximate formula, giving—

$$d_t = \frac{d_0}{1 + ct} = d_0 \frac{1}{1 + ct} = d_0 (1 - ct).$$

Also corresponding to formula (2) of Art. 29 we have—

$$\frac{d_t}{d_t'} = \frac{V_t'}{V_t} = \frac{V_0 (1 + ct')}{V_0 (1 + ct)} = \frac{1 + ct'}{1 + ct} = 1 + c (t' - t).$$

---

* See *Matriculation Hydrostatics*, Chap. ii.

The coefficient of expansion of a substance may thus be deduced from values of its density at different temperatures. This proves of great use in the case of liquids, for the density of a liquid is often more easily ascertainable than its volume.

Example. The density of a piece of glass at 10° C. is 2·6001, and at 60° C. it is 2·5967. Find the mean coefficient of volume expansion of glass.

Applying

$$\frac{d_t}{d_{t'}} = 1 + c\,(t' - t)$$

we get

$$\frac{2{\cdot}6001}{2{\cdot}5967} = 1 + 50c$$

$$\therefore\ 50c = \frac{34}{25967}$$

$$\therefore\ c = \frac{68}{2596700} = {\cdot}000026.$$

EXERCISES IV.

1. Explain any method of measuring the expansion produced in rods by heating.

2. Describe in detail some device for keeping constant the rate of a clock in spite of variations of temperature.

3. The rim of the balance-wheel of a watch is made up of rings of two different metals, one outside the other, and is cut at two points. Explain how it is possible for the rate of the watch to be the same in hot and cold weather.

4. Two rulers, the one of iron, and the other of brass, are placed one upon the other and riveted together so as to form a single ruler of two layers. At a temperature of 60° the compound ruler is straight. Describe and explain what occurs when it is successively exposed to a temperature of 30° and to a temperature of 100°.

5. The coefficient of linear expansion of brass is 0·000019. What will be the volume at 100° C. of a mass of brass the volume of which is exactly 1 cubic decimetre at 0° C.?

6. Explain what you mean when you say that the coefficient of linear expansion of brass is 0·000018.

If a brass yard measure be correct at the temperature of melting ice, what will be its error at the temperature of boiling water?

7. Describe an accurate method of determining the mean coefficient of linear expansion of a metal rod between two given temperatures.

8. What is meant by the statement: The coefficient of linear expansion of copper is ·000017?
If a copper rod is 20 yards long at 0° C., how much longer will it be at 90° C.?

9. The length of an iron rod at 0° C. is 100 cms. Find its length at 10° C., the mean coefficient of linear expansion of iron being ·000012.

10. The length of a copper rod at 10° is 200·034 cms. Find its length at 100° C., the mean coefficient of linear expansion of copper being ·000017.

11. Show that the mean coefficient of superficial expansion for a given substance is approximately equal to twice the mean coefficient of linear expansion for the same substance.

12. A light lath a metre long can turn about a hinge at one end. A piece of wire two metres long is attached to the lath at a distance of one centimetre from the hinge. The wire is vertical and its upper end is fixed in such a way that the lath is horizontal. On heating the wire 50° C. the end of the lath sinks through 16 centimetres. Find the coefficient of expansion of the wire.

13. The coefficient of linear expansion of steel is said to be ·000012. What do you mean by this? If a steel yard-stick be correct at the temperature of melting ice, what is its error at the temperature of boiling water?

14. A brass and a steel rod are each one metre long at 10° C.; find the difference in their length at 60° C.

15. A platinum wire is found to be 0·013 cm. longer at 60° C. than at 40° C. Find the length of the wire at 0° C. Why can a platinum wire be easily fused into a glass tube?

16. Suppose a substance has a coefficient of linear expansion ·003, would it be right to say that its coefficient of cubical expansion is exactly ·009? If not, why?

17. A lump of iron has a volume of 5·5 cubic ft. at 100° C. Find its volume at 25° C., the coefficient of linear expansion of iron being ·000012.

18. The volume of a piece of glass at 100° C. is 100·258 c.cms., and its volume at 0° C. is 100 c.cms. Find the mean coefficient of cubical expansion of glass between 0° C. and 100° C., and thence deduce approximately the mean coefficient of linear expansion between the same limits of temperature.

19. The volume of a mass of lead at 50° C. is 50 c.cms., and at 80° C. its volume is found to be 50·126 c.cms. Show that the mean coefficient of cubical expansion of lead between 50° C. and 80° C. is approximately 0·000084.

20. A glass ball at temperature 40° C. is lowered into a vessel full of water at 40° C., and one pint of water overflows. Find what quantity will overflow if the same glass ball at temperature 100° C. is placed in a vessel full of water at 100° C. The coefficient of cubical expansion of glass is ·000025.

21. Compare the density of lead at 100° C. with its density at −100° C., assuming its coefficient of expansion to remain constant within these limits of temperature.

22. Find the mass of a cubic centimetre of silver at 250° C., the density of silver at 0° C. being 10·31 grammes per c.cm.

23. Compare the densities of platinum and silver at 100° C., given that the densities of platinum and silver at 0° C. are 21·500 and 10·570 respectively.

CHAPTER V.

EXPANSION OF LIQUIDS.

41. Since a body of liquid or gas has no inherent shape but always takes up the shape of the vessel containing it, we cannot speak of a liquid or gas possessing a linear or a superficial expansion. We can deal only with cubical or volume expansion. And it is this expansion which is called *the* expansion of the liquid or the gas.

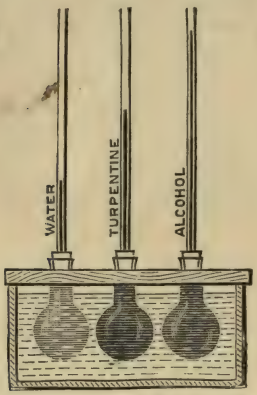


Fig. 24.

Having established that liquids expand on heating, it is of interest to find out whether, as in the case of solids, different liquids differ in expansibility. This point may be settled by a simple experiment.

Exp. 12. Fit up three exactly similar and equal flasks as for Exp. 2, and fix them in holes in a board (Fig. 24). Fill one with water, another with turpentine, the third with alcohol, and, by pushing the corks in more or less, adjust them so that the liquids rise to the same height in each tube. Then put them all into a vessel of warm water. The column of liquid in each tube will be observed to rise,*

* If suddenly immersed in hot water the columns may suddenly sink at first. This is due to the flasks expanding before the heat reaches the liquids.

but the amount of the rise will be different for each liquid. This indicates that each liquid is more expansible than glass, and also that the liquids differ between themselves in expansibility. The order of expansibility for the liquids chosen will be found to be water, turpentine, alcohol, the last being the most expansible.

42. Real and apparent expansion. When the expansion of a liquid is observed in a graduated containing vessel, the expansion which is *apparent*, or which may be observed, is the combined result of the real expansion of the liquid and the expansion of the containing vessel. If the liquid is more expansible than the material of the vessel it appears to expand, but if the liquid is less expansible than the vessel (as in the case of water between 4° C. and 6° C. in a glass vessel) it appears to contract, and if it were possible to find a liquid which expanded equally with the material of the vessel there would be no apparent expansion. Hence the expansion of a liquid indicated or made *apparent* in a vessel initially graduated in equal divisions is known as the *apparent expansion* of the liquid in that vessel.

43. The mercury-in-glass thermometer. The graduation of an ordinary mercury thermometer is based upon the apparent expansion of mercury in glass. The degree divisions of the stem bore are all equal in volume *when the stem is all at one temperature*, but if the thermometer is heated from 0° C. to 100° C. the expansions for each degree rise of temperature, although *apparently* equal, really increase as the temperature rises. As the thermometer is heated, say, from 0° C. to 1° C., the mercury must first expand so as to fill up the increase in the volume of the bulb and stem up to the 0° C. mark due to this rise of temperature, and then fill the degree division between the 0° C. and 1° C. marks at a temperature of 1° C. When the thermometer is heated, say, from 20° C. to 21° C., the mercury must first expand so as to fill the increase in the volume of the bulb and stem, not merely up to the 0° C. mark, but up to the 20° C. mark for 1 degree rise of temperature, and then fill the degree division between the 20° C. and 21° C. marks, not at a temperature of 1° C.,

but at a temperature of 21° C. The real expansion corresponding to the degree rise of temperature from 20° C. to 21° C. is therefore greater than that corresponding to the degree rise of temperature from 0° C. to 1° C. If, however, the volume of the bulb is large compared with that of the graduated stem, it may be assumed that the mercury has to expand the same amount per degree rise of temperature, that is that our scale of temperature depends upon the apparent expansion of mercury in a glass vessel, and other liquids are said to expand uniformly or not according as their apparent expansions in glass vessels are exactly proportional to those of mercury.

Recent work has shown that it is preferable to take our standards of temperature from a gas hydrogen thermometer, but since the expansion of mercury in a glass vessel is very nearly proportional at all ordinary temperatures to that of the expansion of hydrogen in a vessel of the same glass no harm results in adhering to the scale of temperature of a mercury thermometer in ordinary work.

Exp. 13. Find how the apparent expansion of water depends upon the temperature. Take the flask and tube containing water in Exp. 2 and immerse it in a mixture of ice and water. As the water in the flask cools the indicating column will be observed to fall for some time, then to become stationary for a few seconds, and finally to rise, showing that at a certain temperature water is at its maximum density, *i.e.* that if water at this temperature be either heated or cooled it will expand.

Reverse the experiment by withdrawing the now ice-cold flask from the ice and holding it in the air or in warm water. Observe that the temperature of maximum density is much nearer the freezing point than the boiling point.

Of course the above irregularity of expansion may be due to the glass and not to the water. If, however, the experiment be repeated with other liquids no such irregularity occurs; hence it must be due to the water.

44. Coefficient of real and apparent expansion. It was shown above than when a liquid contained in a vessel was heated the observed or *apparent* expansion was less than the *real* expansion. To find the relation between the coefficients of real and apparent expansion of a liquid and the expansion of the solid composing the vessel, let us consider

the case of a liquid contained in a glass vessel which was graduated at 0° C., *i.e.* the volumes indicated by the graduations are correct at 0° C. Let V_0 = volume of liquid at 0° C., V_a = the indicated volume at t° C., and V_t the true volume at this temperature. Let c_a, c_r be the apparent and real coefficients of expansion and c the coefficient of cubical expansion of the vessel.

The real expansion of the liquid = the product of the volume at 0° C., the coefficient of real expansion, and the temperature, *i.e.*

$$V_t - V_0 = V_0 c_r t \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly the apparent expansion of the liquid is given by

$$V_a - V_0 = V_0 c_a t \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The difference between them is the volume which a volume V_a of glass at 0° C. has expanded in being heated to t° C. This is equal to $V_a g t$.

$$\therefore V_0 c_r t - V_0 c_a t = V_a g t$$
$$= V_0 (1 + c_a t)\, g t \quad \text{(from 2)}.$$

Dividing throughout by V_0 and t we get

$$c_r - c_a = (1 + c_a t)\, g.$$

Therefore neglecting $c_a t$, which is usually very small compared with unity, we get

$$c_r - c_a = g$$
$$i.e.\ c_r = c_a + g.$$

That is—the mean coefficient of real expansion of a liquid is approximately equal to the sum of the mean coefficient of apparent expansion of the liquid in any vessel and the mean coefficient of volume expansion of that vessel.

The above applies directly to volume expansion, but it is also applicable to linear expansion. For instance, if a brass rod is measured by a steel scale (correct at 0° C.) at two temperatures the coefficient of real linear expansion of the brass can be found by adding to the coefficient of apparent linear expansion of the brass the coefficient of linear expansion of the steel.

It is an easy matter to find the coefficient of apparent expansion, but not so easy to find that of real expansion.

Now the coefficient of real expansion of mercury has been accurately measured in some classical experiments by Dulong and Petit, and by Regnault (see Art. 47). Assuming the result (·0001815) which they found and finding for ourselves the coefficient of apparent expansion of mercury in a glass vessel, we can deduce the coefficient of expansion of the vessel. Then if we measure the coefficient of apparent expansion of any other liquid in the same vessel, we can, by using the formula, calculate the coefficient of real expansion of the liquid.

45. Methods of finding the coefficient of apparent expansion of liquids. There are two easy methods :—

(1) The Weight Thermometer method.

(2) The Volume Dilatometer or Thermometric method.

(1) In practice the **weight thermometer** is simple.

Principle of the method: If a flask full of liquid at 0° C. is heated to $t°$ C., some of the liquid overflows. On cooling, the liquid retreats, but, if heated again to $t°$ C., it will again exactly fill the flask. Suppose m gms. overflow during heating from 0° C. to $t°$ C., and M gms. remain within at $t°$ C. Then, neglecting the expansion of the vessel, the volume of $M+m$ gms. at 0° C. is the same as the volume of M gms. at $t°$ C., *i.e.* the apparent increase in volume of M gms. of liquid when heated from 0° C. to $t°$ C. is equal to the volume which m gms. occupy at 0° C. Therefore, denoting by V_0 and V_t the volume of M gms. at 0° C. and $t°$ C. respectively, we have

$$\frac{V_0}{V_t} = \frac{M}{M+m},$$

and therefore c_a, which is equal to

$$\frac{V_t - V_0}{V_0 t}$$

$$= \frac{(M+m)-M}{Mt} = \frac{m}{Mt}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

i.e.

$$c_a = \frac{\textit{mass of liquid expelled}}{\textit{mass left in at the higher temperature} \times \textit{change in temperature}}.$$

Practice of the method: This is best illustrated by an experiment.

Exp. 14. Make a weight thermometer and use it to find the coefficient of apparent expansion of mercury. Make a glass vessel 3 ins. long of the form shown in Fig. 25 from a piece of clean and dry thin ½-inch glass tubing. The stem should have a fine bore, but should be strong enough to bear the weight of the thermometer when full of mercury. Weigh it. Placing all the apparatus on a mercury tray, support the tube in a wire-gauze basket, with the neck dipping under some hot mercury contained in a small dish (as illustrated). Fill the tube with clean and dry mercury by alternately heating and cooling. It is quite likely that the first two or three tubes will smash during this operation, but finally one will pass safely through the ordeal. Place the tube, with its opening still under mercury, in the balance case with a Centigrade thermometer beside it. Leave for fifteen minutes. Then read the temperature t, and, hanging the tube upon the hook of the balance, quickly weigh it, taking care that no mercury oozes out through the opening in the meanwhile. (The balance case may conveniently be slightly cooled by placing a beaker of iced cold water in it during this operation.)

Fig. 25.

Now introduce the tube into a beaker of water so that only the narrow part projects outside; heat the water to boiling (100° C.), and keep it so for five or ten minutes. Note that mercury oozes out of the point. Do not waste this mercury: collect it and put it aside.* Remove the tube carefully, wipe it, and cool it, taking care that no hot water enters the narrow neck as the mercury contracts on cooling. Reweigh. From the three weighings find

M, the mass of mercury left in the tube after heating to 100° C.,

m ,, ,, ,, ,, expelled during the heating from t_1° C. to 100° C.; then

$$c_a = \frac{m}{M\,(100 - t_1)}.$$

The mercury may be emptied out now and the experiment repeated with different liquids.

---

* Instead of weighing the tube before heating, the mercury that oozes out may be collected in a small crucible and weighed separately. By weighing such a small mass a more accurate value of m may be obtained than if m is obtained as a difference of two large masses. If this is done it is easy to start the experiment from 0° C.

Example. A weight thermometer when empty weighed 5·913 gms., when full of mercury at 12° C. 61·755 gms., when full at 100° C. 61·022 gms. Find the coefficient of linear expansion of the glass of which it is composed.

$$m = 61{\cdot}755 - 61{\cdot}022 = {\cdot}733 \text{ gm.},$$
$$M = 61{\cdot}022 - 5{\cdot}913 = 55{\cdot}109 \text{ gms.},$$
$$t_1 = 12° \text{ C.};$$
$$\therefore\ c_a = \frac{m}{M\,(100 - t_1)} = \frac{{\cdot}733}{55{\cdot}109 \times 88} = {\cdot}000151.$$

Regnault found that the coefficient of real expansion of mercury is ·000181; hence the coefficient of cubical expansion of the specimen of glass used in the given weight thermometer is given by

$$c = c_r - c_a$$
$$= {\cdot}000181 - {\cdot}000151 = {\cdot}000030.$$

Hence l, the coefficient of linear expansion of the glass,

$$= \frac{{\cdot}000030}{3} = {\cdot}000010.$$

If c_a be considered known, then the above data, and the relation between them expressed by (3), may be employed for the determination of temperature. For, we have—

$$t = \frac{m}{Mc_a}.$$

Thus the process described above might have been employed to determine t.

Fig. 26.

Fig. 27.

The weight thermometer can thus be used as a true thermometer; as such it has the great disadvantage that a difficult experiment is necessary for the determination of

each individual temperature. It may be conveniently used as a maximum thermometer, for the value of t given by the above equation always represents the maximum temperature experienced.

Other forms of the weight thermometer are the specific gravity bottles (Fig. 26) and the pyknometer (Fig. 27). These can be easily made from glass tubing, and have the merit of being easily cleaned and filled.

(2) The volume dilatometer or thermometric method.

Principle of the method: A bulb is blown on a piece of very uniform thermometer tubing. The volume of the bulb and the cross section of the tube are found and the tube can then be graduated and the volume up to any graduation mark ascertained. The bulb and part of the stem are filled and the volume at 0° C. found. The dilatometer is then placed in a bath and its temperature raised. The temperatures at which the liquid column in the stem reaches the various marks is found. The increases of volume can thus be calculated and finally

$$C_a = \frac{\textit{Increase of volume}}{\textit{Volume at } 0^\circ \text{ C.} \times \textit{change of temperature}}.$$

Practice of the method: This is best illustrated by an experiment.

Exp. 15. Make a volume dilatometer and use it to find the coefficient of expansion for water at various temperatures. Blow a bulb $1\frac{1}{2}$ ins. in diameter on a piece of capillary tubing of $\frac{1}{10}$ inch bore and 20 ins. long (Fig. 28). Make file marks along the stem at intervals of 2 inches. Measure the distances between the first and succeeding marks with a finely divided scale or vernier microscope. Weigh the dilatometer. Slightly warm the bulb, immerse the end of the stem in mercury and pick up a thread of mercury 6 or 7 inches long. Measure the length with a vernier microscope or a steel scale. Again weigh the dilatometer. The increase in weight divided by the density of mercury at the temperature of the observation gives the volume of mercury. This divided by the length of the column gives the cross-section of the tube at the temperature of the observation. Expel the mercury by warming the tube and then fill the tube with well-boiled distilled water. Arrange so that at ordinary temperatures the water column stands just below the first file mark nearest the

Fig. 28.

bulb: clean and dry the outside and weigh. The increase in weight in grammes is equal the volume of the water in c.cs. at the freezing point.* Measure the distances between the first and succeeding marks with a finely divided scale or vernier microscope. Immerse the bulb in a large beaker of water provided with a good thermometer and a stirrer. Adjust the source of heat (a small flame some distance below) so that the liquid surface remains at the first mark on the stem for several minutes, the water meanwhile being well stirred. Read the temperature. Similarly for second and succeeding marks up to temperatures of 95° C. Tabulate and plot lengths against temperatures. The graph will not be straight; draw a fair curve through the points. Note it is concave upwards, showing that the coefficient of expansion is greater for higher temperatures. Find the mean length expanded per degree from 25° to 50°, 30° to 35°, etc., and then the coefficient of expansion between these temperatures.

Example. A volume dilatometer whose stem is uniformly graduated contains water. At 0° C. the level was at the 6th division. When heated to 10° C. it stood at the 7th division, and when heated to 20° C. at the 12th division. A thread of mercury occupying 24 stem divisions weighs ·072 gm., and the quantity occupying the bulb and stem up to the zero division weighs 12·24 gms. Find the mean coefficient of expansion of water between 20° C. and 30° C.

The ratio of the volume of the space between two stem-divisions to the volume of liquid at 0° C. is evidently ·003 to 12·24, *i.e.* 1 to 4080. The volume at 0° C. is therefore 4086, the volume at 10° C. 4087, and the volume at 20° C. 4092, *i.e.* the expansion between 10° C. and 20° C. is 5 divisions. The mean coefficient of apparent expansion is therefore equal to

$$\frac{4092 - 4087}{4086 \times 10} = \frac{1}{8172} = \cdot 000122.$$

The coefficient of volume expansion of glass being taken as ·000026 the coefficient of real expansion of water between 20° C. and 30° C. is ·000122 + ·000026 = ·000148.

46. Constant-volume dilatometers have been devised in which the volume of the bulb is kept constant by introducing into it just so much mercury that the expansion of the mercury is equal to the expansion of the glass bulb. The ratios of the coefficients of real expansion of mercury and that of glass being $\frac{\cdot 000181}{\cdot 000026}$ or as $\frac{7}{1}$ it is evident

* Not exactly. It is the volume at 4° C., but scarcely any in accuracy is introduced by taking it as at 0° C.

that if *one-seventh* of the bulb is filled with mercury the condition of constant volume is obtained.

47. Determination of the real expansion of mercury. The methods adopted, for the determination of this constant, by **Dulong and Petit** and by **Regnault**, were both based on the same principle, and differed only in experimental detail.

The method of Dulong and Petit. The principle of this method is simple. If we have two vertical tubes, communicating by a horizontal branch, and each contain a liquid of given density, the junction of the two fluids being in the horizontal branch, then, for equilibrium, the pressures due to the liquid columns on each side of a section, taken anywhere in the horizontal branch, must be equal. Thus, consider the pressures on the section $a\,b$ in Fig. 29. Let h and h' denote the depth of the centre of gravity of the section below the surfaces of the liquid in T and T′ respectively. Then we have, from hydrostatical principles,* that the pressure on $a\,b$ due to the column in T $= h\,d\,g$,† and that due to the column in T′ $= h'\,d'\,g$, where d and d' are the densities of the liquids in T and T′ respectively, and g the acceleration due to gravity. But, since $a\,b$ is in equilibrium, we must have

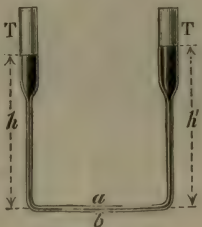


Fig. 29.

$$h\,d\,g = h'\,d'\,g,$$

That is—

$$h\,d = h'\,d'.$$

It is evident that this is true whatever shape a may have, *and whatever be the shape and size of T and T′.*

* See *Matriculation Hydrostatics*, Chap. xi.

† The total force on $a\,b = h\,d\,a\,g$, where a is its area. Pressure is equal to force per unit area, therefore the average pressure over $a\,b = h\,d\,g$.

If, now, the tubes contain mercury, and one column, T, be kept at 0° C., while the other, T′, is raised to a temperature t° C., we have—

$$h_0 d_0 = h_t d_t.$$

$$\therefore \frac{h_t}{h_0} = \frac{d_0}{d_t} = 1 + ct. \qquad \text{(Art. 40.)}$$

$$\therefore \frac{h_t}{h_0} - 1 = ct.$$

Or—

$$c = \frac{h_t - h_0}{h_0 t}.$$

Thus, the mean coefficient of real expansion can be determined by noting $(h_t - h_0)$, h_0, and t, **and the result is quite independent of the expansion of the tubes containing the mercury.**

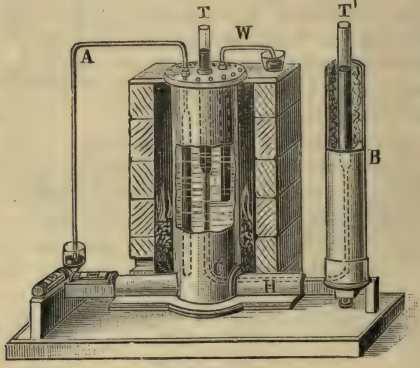


Fig. 30.

The apparatus used by Dulong and Petit is shown in Fig. **30**. The tube T′ was enclosed in a cylinder, B, and surrounded with melting ice. T was placed in a thick copper cylinder, C, which was filled with oil; and, for the purpose of heating it, a furnace was built round it. This is shown in section in the figure. The temperature of this oil bath was given by two thermometers—a mercury weight thermometer, W, and an air thermometer, A. The air

thermometer will be described in the next chapter; it is, however, interesting to notice that this is the first time air was used as a thermometric substance to the exclusion of mercury. Dulong and Petit remarked that, at high temperatures, the indications of the two instruments did not agree; and so they decided to base their calculations on the indications of the air thermometer.*

The heights of the columns of mercury were read by means of the cathetometer,† which was invented expressly for this purpose. The level of the mercury in T was first read, then the level in T', and finally the level of the *axis* of the horizontal tube, H. The difference of the first two readings on the cathetometer scale gave $h_t - h_0$, and the difference of the last two readings gave h_0. The temperature $t°$ C. of the oil bath, and therefore of the mercury in the tube T, was given by the air thermometer A. Thus all the data for calculating the mean coefficient of real expansion of mercury were determined.

The chief results obtained by Dulong and Petit were:—
The mean coefficient of real expansion of mercury

between 0° and 100° = 0·0001818;
,, 0° and 200° = 0·0001843;
,, 0° and 300° = 0·0001887.

This shows that the mercury expands faster at higher temperatures.

*It is important to notice that if Dulong and Petit had used only a *mercury* thermometer, they would not have detected any irregularity in the expansion of *mercury*.

†This instrument consists essentially of a vertical brass or steel scale, to which is attached a horizontally fixed telescope, capable of motion up and down the scale. To determine the vertical distance between any two points, the telescope is focussed on the higher point, and its position on the vertical scale read off. It is then lowered and focussed on the lower point, and the corresponding scale reading again taken. The difference of the two readings thus obtained gives the required vertical distance between the points. The telescope is furnished, in its field of view, with two fine cross wires at right angles to each other, and, in focussing, its position on the scale is so adjusted that the point viewed is made to coincide with the intersection of these cross wires.

Regnault made a very careful determination of the real expansion of mercury by this method with a greatly improved form of apparatus, shown diagrammatically in Fig. 31. The tubes, A C, F N, were joined together by a horizontal tube, B G, at a point, O, along the top of which was made a fine pin-hole. The horizontal tube at the bottom of Dulong and Petit's apparatus was replaced by two bent tubes, C D E, N K L, terminating in a tube, M, which led to an air reservoir. Mercury was poured in the tubes at A and F. Pressure was applied to M to keep the mercury levels in D E and K L about half-way up the tubes. When the mercury overflowed at O, the tubes were ready for heating. The dotted lines represent the heating baths. Each bath was well provided with stirrers. The temperature of the hot bath was measured by an air thermometer, T, and that of the cold baths by mercury thermometers, *t*, *t*, *t*. As the heating proceeded, the mercury level in K L rose above that in D E, showing that hot mercury is less dense than cold mercury. The vertical distances between the hole, O, and the centre of the tubes, C D, K K, and the heights of the columns, D E, K L, above the same centres, were now accurately measured.

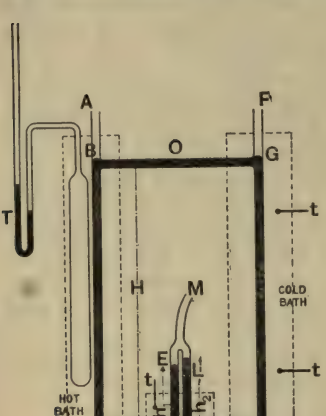


Fig. 31.

Let H = distance from O to centre of tubes, C D, K H; h_1, h_2 = heights of columns, D E, K L, respectively; T = temperature of B C and d_T, the density of the mercury at this temperature; t = temperature of G N, K L, D E, and d_t the density of the mercury at this temperature.

Now, the air in M presses on the top of the columns in D E, K L with the same pressure, also the atmospheric pressure is the same on the top of the columns, B C, G N, ∴ the mercury pressure at E = mercury pressure at L,

$$\therefore \mathrm{H}d_T g - h_1 d_t g = \mathrm{H}d_t g - h_2 d_t g$$

or

$$\mathrm{H}d_T = (\mathrm{H} + h_1 - h_2)\, d_t.$$

Let c_r denote the coefficient of real expansion of mercury, then expressing the densities in terms of density at 0° C., viz. d_0, we get

$$\mathrm{H}\frac{d_0}{1 + c_r \mathrm{T}} = (\mathrm{H} + h_1 - h_2)\frac{d_0}{1 + c_r t},$$

$$\therefore 1 + c_r \mathrm{T} = \left(\frac{\mathrm{H}}{\mathrm{H} + h_1 - h_2}\right)(1 + c_r t).$$

Now $c_r t$ is small, hence in the term $(1 + c_r t)$ Regnault could use Dulong and Petit's value of c_r without vitiating his result for c_r obtained by this formula.

The great advantage of this method is that the heights to be measured are very nearly in the same vertical line, and so cathetometer errors are minimised.

Regnault made over 120 experiments between temperatures of 0° C. and 350° C. He plotted his results on a curve, and from the curve he calculated that the mean coefficient of real expansion of mercury between

0° C. and 100° C. = ·0001815,
0° C. and 200° C. = ·0001841,
0° C. and 300° C. = ·0001866.

That is, if the temperature scale be derived from the air thermometer, the coefficient of expansion of mercury slightly increases with rise of temperature.

The increase is, however, very little at ordinary temperatures, hence our justification for the use of the mercury thermometer in all ordinary work.

Exp. 16. Find the coefficient of real expansion of a liquid, say mercury, by a modification of Regnault's experiment. Take some ¼ inch glass tubing, clean it and dry it, and then bend it into two U tubes, A and B (Fig. 32), making the lengths as long as possible —2 feet if possible, but at any rate not less than 1 foot. Choose two suitable lengths of 1″ glass tubing, fit them with corks, and bore

the corks for the tubes and thermometers as shown. (If the thermometers may be tied on to A and B, no holes through the corks are required for them.) Carefully thread A and B through the corks, provide the side tubes and the thermometers, and mount the whole on a wooden upright, or tie the tubes to two vertical retort stands. It will be necessary to provide some scales. If millimetre scales on slips of porcelain or glass mirror are to be had, tie them on to A and B, If not, place wooden millimetre scales vertically at the back of the large tubes, and a vertical looking-glass behind them. or the heights may be read by a cathetometer.

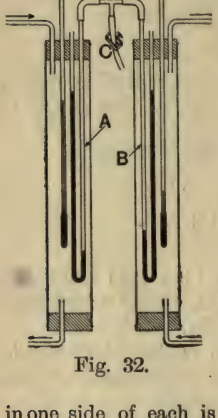


Fig. 32.

Pour clean mercury, little by little, into A and B. It any bubbles of air remain in the columns push a long wire down through and dislodge them. Connect the tops of A and B by thick rubber tubing to a glass T-piece, the remaining end of which communicates by another piece of thick tubing to a bicycle pump. This piece of tubing should be provided with a screw clip. Pump air into A and B until the mercury in one side of each is depressed nearly to the bottom, and the mercury in the other side elevated nearly to the top. Screw the clip tight. Read the differences of level of the two columns in A and the same in B. They should be equal, and should remain constant. If not equal the scales are at fault and must be readjusted. If the heights decrease there is a leakage and the rubber tubing must be tied on. Now pass cold water (ice-cold by preference) through B's jacket, and hot water at a constant temperature, or steam, through A's jacket. The mercury gradually acquires the surrounding temperatures. When it has finished expanding read carefully the difference in levels h_2, in A, and the difference in levels h_1, in B.

Let t_2, t_1 be the temperatures of the hot and cold jackets, and d_2, d_1, d_0, the densities of mercury at these temperatures, and at 0° C. Then equating pressures as before, we have

$$h_2\, d_2 = h_1\, d_1$$

$$h_2\, \frac{d_0}{1 + c_r\, t_2} = h_1\, \frac{d_0}{1 + c_r\, t_1},$$

$$\therefore \quad h_2 + h_2\, c_r\, t_1 = h_1 + h_1\, c_r\, t_2,$$

$$\therefore \quad c_r = \frac{h_2 - h_1}{h_1\, t_2 - h_2\, t_1}.$$

Example. In an experiment with mercury, when a stream of ice-cold water was passed around B and a current of steam around A, the difference of levels in B and A were 410 and 417·5 mm. respectively. Find the coefficient of real expansion.

From the formula we get

$$c_r = \frac{7{\cdot}5}{410 \times 100} = 0{\cdot}000183.$$

More accurate results will be obtained by this method in the case of liquids, such as petroleum and turpentine, which have a large coefficient of expansion.

MEAN COEFFICIENTS OF REAL EXPANSION OF LIQUIDS

between 0° C. and 40° C.

Mercury*	·000181	Turpentine	·00105
Water†	·000193	Ethyl alcohol... ...	·00108
Glycerine	·00053	Chloroform	·00140
Aniline	·00091	Carbon bisulphide ...	·00147
Petroleum	·00099	Ether	·00215

To make these figures comparable with those on p. 41, the former must be multiplied by 3, because they denote linear coefficients, and these refer to cubical.

Harder Example. Determine the ratio between the height of the mercury in the glass vessels of a mercurial pendulum (p. 45) and the length of the steel rod in order that the distance between the top of the rod and the centre of mass of the mercury may be constant at all temperatures.

If the temperature rises from 0° C. to t° C. the volume of the mercury increases in the ratio $(1 + {\cdot}000181\,t) : 1$ while the area of cross section of the tube increases in the ratio $(1 + {\cdot}000009\,t)^2 : 1$. The height of the mercury in the vessel therefore increases in the ratio

$$\frac{1 + {\cdot}000181\,t}{(1 + {\cdot}000009\,t)^2} : 1, \textit{ i.e. } 1 + ({\cdot}000181 - {\cdot}000018)t : 1 \text{ or } 1{\cdot}000163\,t : 1.$$

If h is the height of mercury in the vessel at 0° C. then $h/2$ is the distance of its centre of mass from the bottom and $h/2\ (1{\cdot}000163\,t)$ is the distance at t° and $h/2 \times {\cdot}000163\,t$ is the rise of the centre of mass of the mercury relative to the bottom of the tube.

If l is the length of the steel rod at 0° C., $l \times {\cdot}000012\,t$ is the increase of length for t° C. Therefore we must have for exact compensation

$$l \times {\cdot}000012\,t = h/2 \times {\cdot}000163\,t$$

or

$$h = \frac{{\cdot}000012}{{\cdot}000081} \times l = {\cdot}15\,l \text{ about.}$$

---

* See p. 65.

† The coefficient of expansion of water depends very much on the temperature. It increases rapidly as the temperature is raised. See p. 70.

48. Expansion of water. The expansion of water has been the subject of numerous experiments. It is of special interest, not only because water is so generally used in physical operations, but also because it presents noteworthy peculiarities. Amongst others, Pierre, Despretz, Hällstrom, and Mathiessen have studied this question; the first two used the ordinary dilatometer method, and the last two a hydrostatic method. It is, however, best studied by using a constant volume dilatometer.

Exp. 17. Make a large constant volume dilatometer and study the expansion of water. On a piece of (graduated) capillary tubing blow a large bulb. (It is better to blow the bulb on a piece of ½ in. tubing, and seal a piece of (graduated) capillary tubing* on to this.) Weigh the bulb, fill it with mercury and weigh again, and then empty out six-sevenths of the mercury, thus getting a constant volume dilatometer. Find also the cross section of the capillary tube as in Exp. 15. Finally fill up with water to a convenient point in the stem. If the stem is graduated so much the better; if not tie a millimetre scale (paper is preferable) on to the stem and insert the bulb in a large bath of ice and water, provided with an accurate thermometer and a stirrer. Take the reading when the column is steady. Warm the water to 2° C. and keep it there for ten minutes. Stir well and take the reading. Repeat for every 2° up to 10° C., and for every 5° up to 40° or above. Check the readings by letting the water cool again to zero. Add ice to facilitate this. Take the mean of the two readings at any one temperature. Plot a curve, as in Fig. 33A, showing how the reading varies with the temperature.

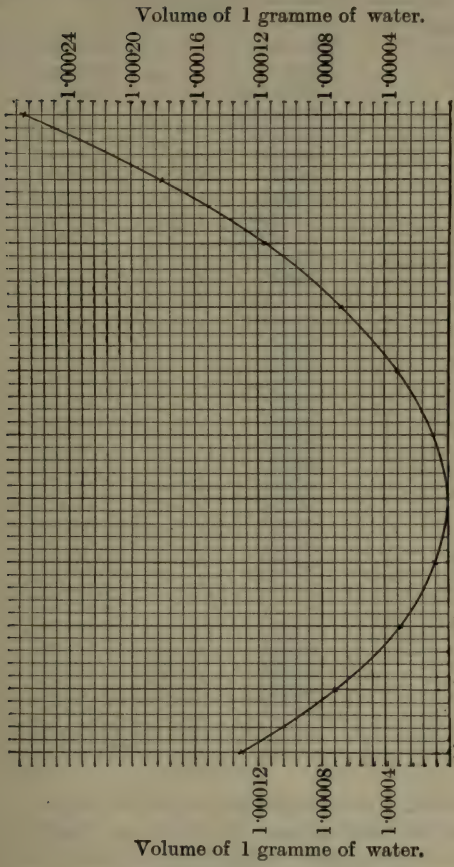
Fig. 33.

Note that as heating proceeds the column falls slowly and then very slowly until a temperature of 4° C. is reached. After this it rises, at first very slowly, and afterwards more rapidly, as long as the temperature is raised. At 4° C. therefore the volume is least, *i.e.* water is at its greatest density at 4° C.

From the smoothed curve and the known dimensions of bulb and stem, calculate the coefficient of *real* expansion of water between

---

* The stem of a broken thermometer will serve the purpose well.



Relation between the volume (of one gramme) of water and its temperature.

Fig. 33A.

various temperatures. Taking the volume of 1 gramme of water at 4° C. as 1·0000 c.c., find the volume at other temperatures. Check your results by the following table:—

Table showing the density of water and the volume of 1 gramme of water at various temperatures.

Temperature Centigrade.	Density.	Volume.
0° (ice)	·9167	1·09081
0°	·999874	1·000127
1	·999930	1·000071
2	·999970	1·000030
3	·999993	1·000007
4	1·000000	1·000000
5	·999992	1·000008
6	·999970	1·000030
7	·999932	1·000068
8	·999881	1·000119
9	·999815	1·000185
10	·999736	1·000265
12	·999537	1·000464
14	·999287	1·000714
16	·998988	1·001013
18	·998642	1·001360
20	·998252	1·001751
25	·997098	1·002911
30	·995705	1·004314
35	·994098	1·005936
40	·99233	1·00733
45	·99035	1·00974
50	·98813	1·01201
55	·98579	1·01442
60	·98331	1·01697
65	·98067	1·01971
70	·97790	1·02260
75	·97495	1·02569
80	·97191	1·02890
85	·96876	1·03224
90	·96556	1·03574
95	·96212	1·03938
100	·95863	1·04315
100 (steam)	·000598	1672·5

With the volume dilatometer water can be lowered much below 0° C. before freezing sets in, and it has been found that expansion continually takes place on cooling until freezing does set in. Even — 20° C. has been reached with liquid water. The slightest jar at this temperature, however, starts solidification and the temperature rises to 0° C.

49. Hope's experiment. Hope devised a simple experiment to determine approximately the temperature of maximum density. A tall glass jar, having two side openings fitted with thermometers, as shown in Fig. 34, is surrounded at its centre with an outer vessel containing a freezing mixture (Art. **90**). The vessel is filled with water at the ordinary temperature, and allowed to cool under the influence of the freezing mixture. As cooling goes on, the lower thermometer falls steadily, while the upper one undergoes but little change; but as the temperature of the former approaches 4° C., it falls more and more slowly, and finally becomes stationary; meanwhile, the upper thermometer begins to fall more rapidly, and continues to do so until 0° C. is reached. Ultimately, a thin crust of ice begins to form on the surface of the water, and the lower thermometer remains stationary at 4° C. These facts admit of simple explanation. The freezing mixture cools the water nearest it, this contracts, becames denser, and descends, causing the lower thermometer to fall rapidly. This goes on so long as a fall of temperature causes the water to contract, but below 4° C. expansion sets in, and then the water cooled by the freezing mixture rises instead of falls, and causes the rapid fall of the upper thermometer to 0° C., while the lower thermometer remains stationary at the temperature of the lowest, and therefore densest, layer; and its reading consequently indicates, approximately, the temperature of maximum density of water. This experiment illustrates what actually takes places in pools of water during frosty weather; the surface freezes, but the temperature of the deeper layers of water seldom falls below 4° C., and thus the lives of fish and other aquatic animals are preserved. It should be noticed here that water expands on freezing, and thus the ice floats on the surface. Were this not the case, each layer of ice would sink as it was formed, and ultimately all the water in the pools would be converted into ice.

Fig. 34.

Exp. 18. Fit up a tin can (instead of the glass jar above) and carry out Hope's experiment. As a freezing mixture use ice and salt.* Take readings every 10 minutes. Plot curves, with temperatures as ordinates and times as abscissae. The curve for the lower thermometer will show a long horizontal portion indicating a temperature very nearly 4° C.

50. Barometric correction for temperature. The pressure of the atmosphere is usually expressed in terms of the height of a column of mercury at 0° C. which exerts an equivalent pressure.† The *observed* height of a barometer at $t°$ C. has to be reduced to the *equivalent* height at 0° C. ; this is called correcting for temperature, and in applying the correction two points are to be remembered : (i) To correct for the expansion of the scale between 0° C. and $t°$ C. ; (ii) To correct for the change of density of the mercury. Let H denote the *observed* height of the barometer at $t°$ C., H_0 the true height at 0° C., l the mean coefficient of expansion of the scale, c_r the mean coefficient of real expansion of mercury, and t the temperature of observation. Then the true height, corrected for the expansion of the scale, is $H(1 + lt)$. Denote this by H_t. Then for (ii) we have, as in Art. 47, $H_t\, d_t = H_0\, d_0$.

That is— $$H(1 + lt)d_t = H_0\, d_0 .$$

$$\therefore\ H_0 = H(1 + lt)\frac{d_t}{d_0}.$$

But— $$\frac{d_t}{d_0} = \frac{1}{1 + c_r t}. \qquad \text{(Art. 40.)}$$

$$\therefore\ H_0 = H\frac{1 + lt}{1 + c_r t} = H[1 + (l - c_r)t].$$

Or, since c_r is usually greater than l, this is more generally written

$$H_0 = H[1 - (c_r - l)t].$$

It should be noticed that the mean coefficient of real expansion of mercury is employed because the correction (ii) is necessary on account of change of density of the mercury. *On no account must the column of mercury be treated as a rod subject to linear expansion.*

Example. A barometer with a glass scale reads 755° mm. at 18° C. ; find the reading at 0° C. The coefficient of real expansion of mercury is ·000182, and the mean coefficient of linear expansion of glass is ·0000089.

---

* If performed outdoors on a cold winter's day no freezing mixture is required.

† For a description of the barometer and method of reading it see *Matriculation Hydrostatics*, Chap. xiv. ; Bower and Satterly : *Practical Physics*, Chap. v.

Applying the above formula, we have

$$H_0 = 755\,[1 - (\cdot 000182 - \cdot 000009)\,18]$$
$$= 755\,[1 - \cdot 000173 \times 18]$$
$$= 755\,[1 - \cdot 0031]$$
$$= 755 - 2\cdot 348$$
$$= 752\cdot 7 \text{ mm. nearly.}$$

EXERCISES V.

1. Distinguish between the absolute and the apparent expansion of a liquid, and show that the coefficient of absolute expansion is equal to the sum of the coefficient of apparent expansion and of the coefficient of expansion of the vessel.

2. The coefficient of absolute expansion of mercury being $\frac{1}{5550}$ and its coefficient of apparent expansion being $\frac{1}{6480}$, find the coefficient of cubical expansion of glass.

3. A zinc rod is measured by means of a brass scale, and found to be 1·0001 metres long at 10° C. What is the real length of the rod at 0° C. and at 10° C.? [Mean coefficient of linear expansion of zinc is ·000029 and of brass ·000019.]

4. A rod of copper and a rod of iron placed side by side are riveted together at one end. The iron rod is 150 cms. long, and a mark is made on the copper rod, showing the position of the unriveted end of the iron at 0° C. If at 30° the mark is 0·0255 cm. from the end of the iron rod, what is the coefficient of expansion of copper, that of iron being 0·000012?

5. Describe a method of determining the coefficient of apparent thermal expansion of mercury in a glass tube.

6. A weight thermometer when empty weighed 1·3210 grammes. After filling it with mercury at 0° C., and then heating it to 100° C., ·2351 gramme of mercury was expelled. The thermometer, with the residue of the mercury, when cooled down to the atmospheric temperature weighed 16·3075 grammes. From these data calculate the coefficient of apparent expansion of mercury.

7. From the result in Question 6 and the known absolute coefficient of expansion of mercury (·0001815) calculate the cubical coefficient of expansion of the glass.

8. The same thermometer as in Question 6, after filling with glycerine at 0° and then heating to 100° C., expelled 0·0678 gramme of glycerine, and on cooling down it weighed 2·710 grammes. If the coefficient of cubical expansion of the glass is ·000025, calculate the absolute coefficient of expansion of the glycerine.

9. A weight thermometer weighs 50 grammes when empty, and 710 grammes when full of mercury at 0° C. On heating up to 100° C., 10 grammes of mercury are expelled. Calculate the mean coefficient of cubical expansion of glass, assuming the mean coefficient of real cubical expansion of mercury to be 0·000181.

10. A porcelain weight thermometer weighs 165 grammes when empty and 468 grammes when full of mercury at 0° C. When heated to 300° C., the weight of overflow is found to be 13·464 grammes. Find the mean coefficient of cubical expansion of porcelain between 0° and 300° C., assuming that of mercury to be ·000184 for the same range of temperature.

11. The coefficient of absolute cubical expansion of mercury is ·00018, the coefficient of linear expansion of glass is ·000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101·56 divisions?

12. A mercury thermometer wholly immersed in boiling water reads 100° C. When the stem is withdrawn so that the graduations from 0° upwards are at an average temperature of 10° the reading is 98·6°. Find the coefficient of apparent expansion of mercury in glass.

13. An ordinary mercurial thermometer is placed with its bulb and lower part of stem in a vessel of water, and indicates a temperature T. The upper portion of the stem, containing n divisions of the mercury column, is in the air at a temperature t. What is the true temperature of the water in the vessel?

14. In an experiment made with Dulong and Petit's apparatus to find the expansion of aniline, the column at 0° C. was 62·3 cms. high and the column at 100° C. was 5·67 cms. higher. Find the coefficient of real expansion of aniline.

15. What is meant by the density of ice-cold water? Does the density of ice-cold water change when the water is warmed, and, if so, in what way? By what experiment would you find out whether any change in density occurs under these circumstances?

16. Water is said to have its maximum density at 4° C. Explain what this means.

In what respect is the behaviour of mercury different from that of water when both are gradually warmed from 0° C.?

17. An empty weight thermometer weighed 4·8155 grammes. It was filled with water at 10° C. and gradually heated in a bath. At 20°, 40°, 60°, 80°, 95° it was withdrawn and weighed, the weights being 22·9032, 23·7966, 23·6340, 23·4220, and 23·2376 grammes. Find the coefficient of real expansion between 20° and 40°, 40° and 60°, 60° and 80°, 80° and 95°. Draw a curve illustrating the change of coefficient of real expansion with temperature.

18. If δ be the expansion of water between 4° and 0° C., and Δ its expansion between 4° and $t°$, show what is the density of water at $t°$ referred to water at 0°.

19. A barometer is provided with a silver scale which reads correctly at 0° C. What is the true height of the mercury corresponding to an apparent height of 761·05 mm. at 15° C.? What height would the barometer register at 0° C.?

CHAPTER VI.

EXPANSION OF GASES.

51. In Art. **7** it was shown by a simple form of apparatus (Fig. 3) that gases expand when heated; other simple pieces of apparatus which also illustrate the same fact are shown in Figs. 35-37.

Fig. 35.

Fig. 35 is made by simply blowing a large bulb on a narrow tube. While still hot the open end of the tube is immersed in a liquid—mercury or coloured alcohol. When a column of liquid of sufficient length ($\frac{1}{2}$ in.) has been drawn up, the tube is withdrawn. The thread serves to separate the enclosed air from the outside air, and also to indicate its volume of the enclosed air. When cold the thread of liquid settles at *m*. If now the bulb be slightly warmed the thread rises, showing that the air has expanded; if the bulb be cooled the thread falls, showing contraction.

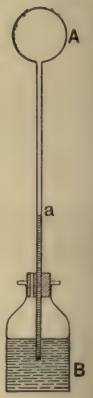


Fig. 36.

Fig. 36 is a modification of the above. The bulb, A, while still hot is thrust through a cork fitting tightly into a bottle, B, containing a coloured liquid (alcohol coloured with majenta dye is very suitable). As the air in the bulb further

cools the liquid rises up the tube. If the bulb be heated the column is depressed. A fine hole should be bored through the cork in B to enable the air in B to communicate with the atmosphere.

Fig. 37 is another modification which may be used if facilities for blowing a bulb are not available. The essential part is a small flask, F, fitted with a well-fitting india-rubber stopper, through which passes a quill tubing dipping into a coloured liquid contained in a bottle below. A cork is also fitted to this bottle. Heat the flask, air bubbles out through the liquid. Now cool F and a column of liquid rises up the tube.

Fig. 37.

The student should make one of the above pieces of apparatus, provide it with a home-made scale and observe its readings at different parts of the day, and at different days throughout the year. Compare its readings with those of a mercury thermometer fixed in its neighbourhood. Make a rough mercury thermometer of the same size as the apparatus of Fig. 35. Place them side by side and compare their readings. Which expands most—mercury or air? It is plain that the air does.

Exp. 19. Fill a long gas jar with water or mercury. Take a long glass tube, closed at one end, and while keeping it vertical force the open end underneath the surface of the liquid. As the tube is depressed, note that the volume of the air in the tube is decreased. (If mercury is used bring the tube to the side of the cylinder in order to see the mercury in the tube.) Lift up the tube and the air returns to its former volume.

Exp. 20. Take another long tube, fill it about three-quarters full with mercury, and measure the length of the air column still left in the tube. This air is at the atmospheric pressure. Now firmly close the tube with the thumb and invert the tube in a basin of mercury, as in making a barometer. Again measure the length of the air column. It is greater than before, *i.e.* the volume has increased. Now the pressure of the air in the tube (*i.e.* the pressure on the air) is equal to the barometric height less the height of the mercury

column in the tube. Read these two heights and find the difference. Show that

Barometric height × former length of air column = difference between barometric height and height of mercury column × final length of air column,

i.e. the product *pressure* × *volume is a constant.*

This is an anticipatory proof of Boyle's Law. See Art. **53.**

Example. 62 cms. of a tube 82 cms. long was filled with mercury and inverted in mercury. The height of the mercury column was found to be 38 cms., and the length of the air column 40 cms. The barometer read 76 cms. Then

Initial volume of air = 20, initial pressure = 76 cms. of mercury.

Final volume of air = 40, final air pressure = (76 − 38) = 38 cms. of mercury.

Observe that $76 \times 20 = 38 \times 40$.

From the above experiments we conclude that **the volume of a given mass of gas depends upon its pressure as well as upon its temperature,** hence in considering the expansion of gases by heat the pressure must not be neglected.

52. The relation between the volume, temperature, and pressure of a gas. If experiments are to be carried out to find the relation between the volume, temperature, and pressure of a given mass of gas, it is obviously best to divide the experiments into three classes, and thus to find

(1) The relation between pressure and volume when the temperature is kept constant.

(2) The relation between volume and temperature when the pressure is kept constant.*

(3) The relation between pressure and temperature when the volume is kept constant.

We shall consider these separately in the following sections :—

---

* This is the simple case of expansion as already studied in the cases of solids and liquids.

53. The relation between pressure and volume: Boyle's Law. In 1662 Boyle stated that **the volume of a given mass of gas is inversely proportional to the pressure when the temperature is kept constant.** This may also be stated as follows:—The product of the pressure and the volume of a given mass of gas at constant temperature is constant, *i.e.*

$$P_1 V_1 = P_2 V_2 = \ldots = \text{a constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Thus if the pressure is doubled the volume is halved, and if the volume is to be increased threefold the pressure must be diminished to one-third its former value.

Exp. 21. To verify Boyle's Law experimentally in the case of air for pressures greater than that of the atmosphere. A piece of apparatus called **Boyle's tube** is generally used (Fig. 38). To make it, take a long piece (say 5 ft.) of stout glass tubing of about $\frac{1}{4}$ in. bore; clean and dry it, and then carefully heat one end and seal it. At about 12 ins. from this end heat the tube and bend it through two right angles. Mount on a board and attach mm. scales to each limb, as in the figure, having the zeros at the same level and as low as possible. Insert a small funnel into the open end for the purpose of pouring in mercury. Take the apparatus into a corner of the room where the temperature will remain constant. As a check hang a thermometer near the short limb. Work over a mercury tray to catch spilt mercury. Pour a little mercury into the bend and adjust by shaking until the mercury surfaces are both at zero. A certain mass of air has now been imprisoned in the short limb at atmospheric pressure. Read its volume, assuming the bore of the tube to be uniform, and taking the volume of unit length of tube as unit volume the number expressing the length of the tube may be regarded as the number expressing the volume. Pour a little mercury into the long limb. Note that the mercury does not rise in the closed limb as rapidly as in the open limb: this is due to the enclosed air exerting pressure. Read the two levels. Pour in more mercury and repeat the readings. Proceed till the long tube is full. Now read the barometer and note its height. The difference in level of the two surfaces in Boyle's tube plus the barometric height is the total pressure exerted on the gas, and therefore exerted by the gas. The volume of the gas is found by

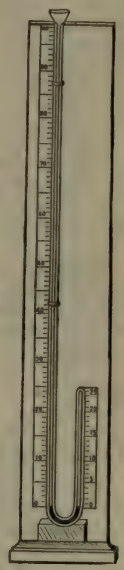


Fig. 38.

subtracting the reading of the mercury surface in the closed limb from the reading of the top of the tube. Enter the results thus: Barometric height = 76 cms. Initial length of air column = 25 cms.

Ht. of Mercury in closed limb. I.	Ht. of Mercury in open limb. II.	Excess Pressure over Atmospheric = I. − II. III.	Total Pressure = Barometric Ht. + III. IV.	Volume of Air deduced from I. V.	Product, Pressure × Volume. VI.
cms.	cms.	cms.	cms.		
0	0	0	76	25	1900
5	24	19	95	20	1900
10	61	51	127	15	1905
...	...	...	...	...	...

The products in Column VI. are practically constant: thus, Boyle's Law is verified.

To verify Boyle's Law for any gas it is only necessary to substitute that gas for air in the closed limb.

Example. Suppose 10 c. ft. of oxygen at atmospheric pressure (15 lbs. per square inch) to be forced into a steel cylinder of 0·6 c. ft. capacity. Required the pressure of the gas in pounds weight per square inch.

Apply $P_1 V_1 = P_2 V_2$;

we obtain $15 \times 10 = P_2 \times 0{\cdot}6$;

$\therefore \quad P_2 = 250,$

i.e. the final pressure = 250 lbs. wt. per square inch.

A more satisfactory form of apparatus is shown in Fig. 39. V is a long graduated glass tube closed at one end. Its open end is tightly fitted with a cork, through which there is a glass T-piece, one arm of which communicates by a long piece of thick indiarubber tubing, I R, to an open reservoir, R, consisting of a piece of wide tubing, and the other carries a stopcock, C. The tube, V, may be fixed anywhere along the upright, P. The reservoir, R, can be raised or lowered by means of a string, S, passing over a pulley, W, fixed at the top of P. The mercury levels may be read by means of a millimetre scale attached to the front part of P. The tap, C,

being closed, V is removed from the board and mercury is poured in R until V and a large part of the tubing is filled. The tubing being tightly clipped, V is placed in position and R hung beside it. To the stopcock C is then attached a long drying tube, containing fused calcium chloride. R is then slightly lowered, and C being carefully opened, dry air rushes in and collects at the top of V. R is then carefully lowered until sufficient air has been collected in V, when C must be closed. For experiments at a pressure greater than atmospheric, V should be near the bottom of the stand, and nearly full of air; for experiments at pressures less than atmospheric, V should be near the top of the stand, and about one-third to one-quarter full of air. To perform the experiment R is shifted to any part of the scale and the string then attached to a stud at the back of P. A short time is then allowed for things to come to rest; the mercury levels in V and R and the volume of air in V are then read. This is repeated for different positions of R, and the results are tabulated as before. A thermometer attached to V should indicate a constant temperature throughout the experiment.

Fig. 39.

Exp. 22. Carry out an experiment with an apparatus like the above. A 50 c.c. burette furnished with a glass tap may be used instead of V. The volume between the 50 c.c. mark and the stopper must first be obtained. The open end is then drawn out to fit the indiarubber tubing, and the burette is fixed with the tap end uppermost and communicating to the drying-tube. By elevating R the burette is filled with mercury, R is then lowered and the burette is filled with dry air.

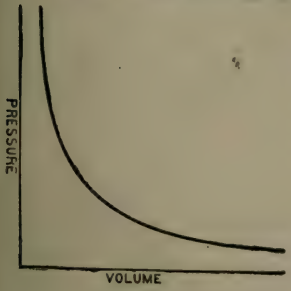


Fig. 39A.

Plot a curve between pressure and volume, pressures being abscissae and volumes ordinates. The curve will be like Fig. 39A. Perfect results would give a rectangular

hyperbola. If the logarithm of the pressure is plotted against the logarithm of the volume the result is a straight line, for since

$$P_1 V_1 = P_2 V_2 = P_3 V_3 = \ldots \text{ a constant}$$
$$\log P_1 + \log V_1 = \log P_2 + \log V_2$$
$$= \log P_3 + \log V_3$$
$$= \ldots$$
$$= \text{a constant.}$$

Plotting therefore log P against log V we get a straight line inclined at 45° to either axis.

54. The relation between volume and temperature, when pressure remains constant. We have learnt that every solid or liquid has its own coefficient of expansion which is always a very small fraction. But all gases have the *same* coefficient of expansion, which is *not* a very small fraction.

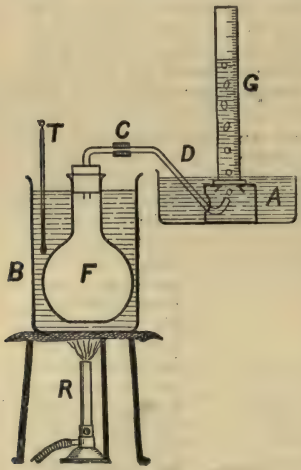


Fig. 40.

Exp. 23. To show that all gases have the same coefficient of expansion. Dry a pint flask, F (Fig. 40), fill it with dry air, and fit it with a cork carrying a stout and short delivery tube, D, leading into a

collecting apparatus, A*. D should be clamped at C, so as to support F. Take a large beaker, B, half fill it with water, and fix it in position. Heat the water very slowly, stirring well, up to 20°, noting the temperature by a thermometer, T. When the temperature has reached 20° C. and bubbles have ceased to escape at the mouth of D, place over it an inverted graduated gas jar, G, of say 200 c.cms. capacity, which has been filled with water. Now heat B till the water boils,† and when the bubbles have ceased to rise note the volume of air collected. Then take the cork out of F and turn off the gas. The volume of air in G is the volume, measured at the temperature of the water in G, by which the quantity of air which filled F at a temperature of 20° C. is increased when heated to 100° C. Repeat the experiment, having previously filled F with coal-gas, hydrogen, or some other common gas. It will be found that the amount of gas which overflows into the measure is the same as was noted in the case of air.

The last experiment is more or less qualitative. The earliest accurate experiments were made by Dalton in England, and by Gay Lussac in France.

55. I. Gay Lussac's method of finding the coefficient of expansion of a gas with pressure. For the purpose of this determination Gay Lussac obtained a large thermometer tube with a spherical reservoir and a long stem. This tube he carefully calibrated and graduated after the plan mentioned in Art. **45**(2). It was then filled with mercury and fitted (as at A, Fig. 41) into a wider tube, T, filled with calcium chloride. By introducing a platinum wire W, into the tube the mercury was slowly shaken out, and its place taken by air which was quite dry owing to it having passed over the calcium chloride in the wide tube. A short column of mercury was left in the stem of the

* This consists of a trough in which is placed a small cylinder, A, with a hole in one side, and another, provided with a conical flange opening inwards, through the top. The trough is filled with water till the cylinder is covered. The delivery tube is passed through the side hole and the gas passes up through the hole in the top.

† To prevent the water bumping as it boils, place a few fragments of unglazed earthenware (broken flower-pot) in the beaker. When boiling occurs, a continuous stream of bubbles of steam will come off from these fragments, the boiling being continuous and comparatively gentle.

tube to act as an index; it was then taken and placed horizontally (as at B, Fig. 41) in a sheet-iron vessel, V, which was first filled with melting ice and afterwards placed on a furnace, F, and gradually heated. The position of the mercury index was noted when the bulb of the tube was surrounded with ice, and also at different temperatures

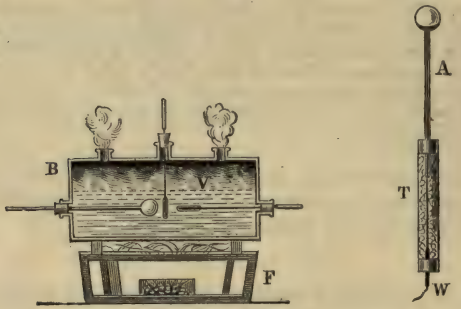


Fig. 41.

as the heating went on. From these observations the mean coefficient of *apparent* expansion of air in glass was calculated, and the mean coefficient of *absolute* expansion deduced, from this result and the known mean coefficient of expansion of the glass tube. It will be seen that this method is identical in principle with that of Art. 45 (2). In this way Gay Lussac found $c_r = 0{\cdot}00375$ or $\frac{1}{268}$. This result we now know to be too high.

Exp. 24. To find the coefficient of expansion of air at constant pressure. Take a piece of tubing, A, of uniform bore (a piece about 2 feet long and $\frac{1}{8}$ inch in bore will do), carefully clean it and dry it, and seal up one end. Heat the tube to over 100° C., and dip the open end in mercury, or strong sulphuric acid. As the tube cools mercury or acid will be forced up into it. Obtain in this way a thread of mercury or acid, P (Fig. 42), about an inch in length. When the tube is cold the thread should be nearly halfway down the tube.

Fasten a thermometer, T, to the tube, and mount the tube by corks in a wide glass tube, G, so that the open end just projects,

and then fix the outer tube horizontally, and provide it with corks, so that a current of water or vapour may be passed through it.

Pass a slow current of ice-cold water (0° C.) through G. By means of a metre scale measure as accurately as possible the distance between the closed end of the tube and the near end of the thread. This length is proportional to the volume of the air at 0° C. Denote this volume by V_0.

Fig. 42.

Now pass a rapid current of steam through the tube. The air in A will expand and force the thread outwards. This expansion will occur at atmospheric pressure ; for the other side of the thread is exposed to the atmosphere. When the thread becomes stationary note again the position of the near end. Denote the volume of the air at 100° C. by V_{100}.

Then c_a, the coefficient of apparent expansion of air in glass at constant pressure, is given by

$$c_a = \frac{V_{100} - V_0}{100 \times V_0}.$$

The coefficient of real expansion of air in glass is of course given by

$$c_r = c_a + c = c_a + \cdot 000025,$$

but since c_a is much larger than c (being about ·00367) and the above experiment would hardly give it correct to the third significant figure it is not worth while correcting for the expansion of the vessel.

Fig. 43.

The above coefficient is the mean coefficient between 0 and 100°. To show that the expansion is uniform, modify the above by bending up the end of the tube and using a long metal water bath, as in Fig. 43. Bind several pieces of copper around the tube at various distances

from the closed end. Heat slowly, stir well, and take the temperatures as the inner end of the liquid column arrives at the wire marks. The temperature should be kept constant for a minute or two before taking the reading. Finally measure the distances of the wire marks from the closed end of the tube. Tabulate and plot lengths and temperatures. The graph is practically straight, showing that the **expansion is uniform.** Deduce from the graph the value of *ca.*

Example. In an experiment like the above, wires were tied around at distances of 36, 38, 40, 42, etc., cms. from the closed end of the tube. The experiment was begun at 0° C., when the near end of the column was between the first and second wires. The temperatures at which it arrived at the other wires were 14°, 29°, 44°, 58°, 74°, 88°. On boiling it did not reach the eighth wire.

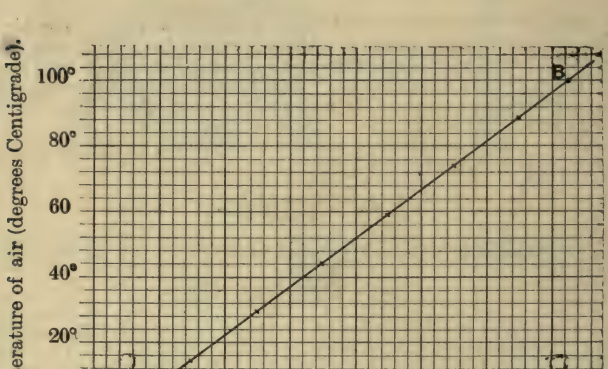


Fig. 44.

Plot the curve (Fig. 44). It is practically straight. Deduce from it that, given by the point A, $V_0 = 36$ cms., and V_{100}, given by the point B, $= 49{\cdot}5$ cms., and hence that *ca*,

which $$= \frac{49{\cdot}5 - 36}{36 \times 100} = \frac{13{\cdot}5}{3600} = {\cdot}00375,$$

II. Regnault's method. In Gay Lussac's method, as described above, there are two important sources of error. (*a*) The moisture clinging to the interior of the thermometer tube was not removed, and, as it was converted into vapour on heating, the volume of the enclosed air appeared to increase more than it really did, hence the result obtained was too high. (*b*) The index of mercury in the stem did not completely separate the enclosed air from the outer air. Regnault modified his apparatus so as to avoid these errors; the interior of the vessel which contained the gas was thoroughly dried by alternately filling it with dry warm gas and exhausting it by means of an air pump, and the mercury index in the stem was replaced by a long column of mercury in a U tube (or manometer). His apparatus is shown in Fig. 45. The reservoir A, in which the gas is placed, is a small spherical vessel which can be enclosed in a suitable metal vessel, M, and the temperature of the contained gas adjusted by filling the space between the two with ice, water, steam, etc. The manometer, B C D E, consists essentially of two vertical tubes communicating below by a metal bend to which a tap is attached. In order to keep it at a uniform and known temperature, it is immersed in a water bath. One of the tubes, B C, ends in a fine piece of tubing which is bent over at B and joined to the stem of A; the other, D E, is much higher. Communicating with

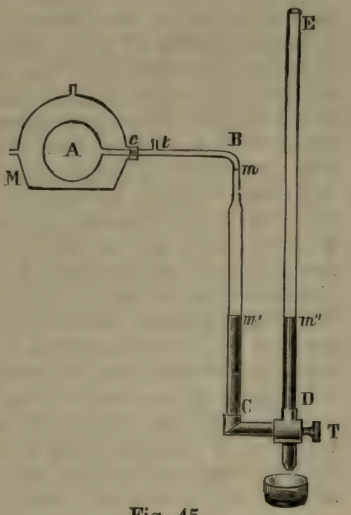


Fig. 45.

the horizontal part between B and A is a small tube, *t*, by means of which the interior of A is dried, and the gas introduced into it after passing through a series of drying tubes. To perform the experiment the reservoir, A, is surrounded with ice, and mercury is poured into the manometer at E until it rises to the mark *m* on the tube, B C, and to a point in D E on the same horizontal line as *m*. This adjustment of levels is readily effected by varying, through *t*, the quantity of air in A. The tube *t* is now sealed off in the blow-pipe, the ice in M removed, and a small quantity of water put in its place and heated till it boils. The reservoir A is thus surrounded with steam at the temperature of the boiling point; the contained gas expands, and, depressing the column of mercury in B C, causes that in D E to rise. The tap T is then opened and some mercury run out, the level in D E falling more rapidly than that in B C. As the gas further expands more and more mercury is run out, and when the expansion is complete the levels of the mercury columns in B C and D E are adjusted in the same horizontal line in a position similar to that shown in the figure at m', m''. The air in A has thus expanded, between 0° C. and t° C., at a constant pressure equal to that of the atmosphere at the time of the experiment,* for, both at 0° C. and t° C., the mercury columns in the manometer have been adjusted so as to have no difference of level. If, then, we know the relative volumes of A, of its stem up to *m*, and of the portion of B C between *m* and m', we can calculate the mean coefficient of absolute expansion in the usual way.

It must be noticed here that the gas in the part of the apparatus outside M is at a different temperature to that in A, hence a correction for this as well as for the expansion of the glass must be applied in working out the result.

Thus, for example, let the initial temperatures of the reservoir and manometer be 0° C. and the final temperatures T° C. and t° C. Also let V_0 and v_0 be the volumes of the reservoir and the space

* If the atmospheric pressure has not remained constant, allowance must be made for this in adjusting the mercury levels at the finish of the experiment.

m m' at 0°.* Then the initial volume of the air at 0° C. is V_0, and the volume of the air if all were at T° C. would be

$$V_0 (1 + c\,T) + v (1 + c\,t) \left(\frac{1 + c_r T}{1 + c_r t}\right),$$

where c_r is the coefficient of real expansion of air and c is the coefficient of volume expansion of glass.

But this must be equal to

$$V_0 (1 + c_r T),$$

$$\therefore V_0 (1 + c_r T) = V_0 (1 + c\,T) + v (1 + c\,t) \left(\frac{1 + c_r T}{1 + c_r t}\right),$$

from which c_r may be calculated.

Regnault found by this method the value 0·0036706 for the mean coefficient of absolute expansion of air between 0° C. and 100° C.; for different gases the value of this coefficient varied slightly, especially in the case of gases which are easily liquefied. Practically, however, this result, which is approximately equal to $\frac{1}{273}$, may be considered as applicable to most gases.

56. Dalton's or Charles's Law. The facts that air and other gases expanded uniformly were first discovered, as mentioned above, by John Dalton, the author of the atomic theory, and afterwards verified by Gay Lussac, and more accurately by Regnault, and their formular expression is known as Dalton's, or Gay Lussac's, or Charles's Law. It may thus be enunciated:—

The coefficient of expansion of any gas under constant pressure is $\frac{1}{273}$, or

If the temperature of any given mass of gas, whose pressure is maintained constant, be raised 1° C., the volume increases by $\frac{1}{273}$ of the volume it would have at 0° C. under the same pressure,

$$i.e.\ V_t = V_0 (1 + c_r t) \ldots\ldots (2)$$

where V_0 = the volume at 0° C., V_t = the volume at t° C. of the same mass of gas, and $c_r = \frac{1}{273}$.

Also

$$V_{t1} = V_0 (1 + c_r t_1),$$

$$\therefore \frac{V_{t1}}{V_t} = \frac{1 + c_r t_1}{1 + c_r t} \ldots\ldots (3)$$

a useful relation.

* V_0 includes the portion of the capillary tube included within the heater, and v_0 includes that portion kept at the temperature of the manometer.

The value of c_r is nearly but not quite independent of the pressure, it increases very slightly with increase of pressure. The value of c_r also varies with the temperature, very little at ordinary and high temperatures, but considerably at low temperatures, especially when the gas is approaching its temperature of liquefaction.

57. Relation between temperature and pressure when the volume is constant. When a gas is heated in a closed vessel so that it cannot expand, it is obvious from our previous knowledge that its *pressure* must increase. We now proceed to find out how it increases in the case of air.

Exp. 25. To find the coefficient of increase of pressure of air at constant volume. Fit up an apparatus as shown in Fig. 46. A is a bulb (2 to 3 ins. in diameter) full of dry air, attached by a stout narrow tube B C D (of $\frac{1}{16}$" bore) bent twice at right angles to a long piece of stout indiarubber tubing, D E F, in the other end of which is thrust a straight bit of open glass tubing, F G. By means of clamps D C is attached to an upright stand, M, and then D E F is filled with mercury so that the mercury rises into D C and F G. F G is attached to a sliding piece, K, which can be fastened at any height by means of a screw-clamp, S. A millimetre scale is pasted on to the front of M, and levelling screws, L, L, L, serve to set M vertical. Take a large beaker, half fill it with cold water and lumps of ice, and fix it so that A is wholly submerged. The temperature of A gradually arrives at 0° C. Adjust K so that the mercury surface in C D reaches a convenient point, *o*, as near the bulb as possible.* Read the level of the mercury surface, *p*, in G F and the height of the barometer. If this is H, then P, the pressure of the air in A, = H *plus* the difference in levels of *o* and *p* if *p* is above *o*, or H *minus*

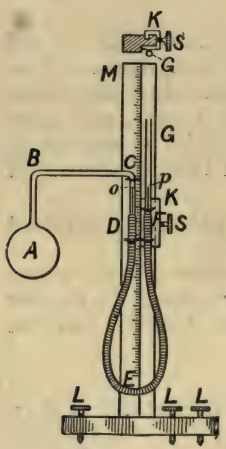


Fig. 46.

* If the apparatus has not previously been set up this may be managed by filling the indiarubber tubing with mercury before slipping it on to the tube, C D. A being previously at the air temperature the mercury will rise up D C as A is cooled to 0° C.

the difference in levels if p is below o. Now put a burner underneath the beaker and heat the water. As the temperature rises the air in A will expand and force the mercury in C D down. Prevent this by lifting up G F so that the mercury is forced back to o. Proceed till the temperature is nearly 20° C., then turn the burner down and keep the bath at 20° C. for some time. When things are steady finally adjust K till the mercury is exactly at o, and then read the height of the mercury surface in G F and the barometer. Deduce P, the pressure at 20° C., in the same way as P_0 was deduced. Now proceed to temperatures of 40° C., 60° C., 80° C., and 100° C. (boiling), and repeat the observations. Tabulate the results.

Plot a curve taking temperatures as abscissae and pressures as ordinates. The result will be a straight line, showing that the pressure increases uniformly with temperature.

i.e. $\dfrac{P_{100} - P_0}{100 \cdot P_0}$, or in general $\dfrac{P_t - P_0}{P_0\, t}$ is a constant.

If the experiment is performed carefully, it will be found that this constant is also equal to $\frac{1}{273}$.

Example. The following shows a set of results which may be obtained.

Temperature	Pressure	Increase of Pressure above 760 mm.	Increase of Pressure per degree rise above 0° C.
0° C.	760 mm.	—	—
21° C.	818 mm.	58 mm.	$\frac{58}{21}$ or 2·79 mm.
32° C.	849 mm.	89 mm.	$\frac{89}{32}$ or 2·78 mm.
54° C.	910 mm.	150 mm.	$\frac{150}{54}$ or 2·78 mm.
75° C.	968 mm.	208 mm.	$\frac{208}{75}$ or 2·77 mm.
100° C.	1038 mm.	278 mm.	$\frac{278}{100}$ or 2·78 mm.

The mean of the last column = 2·78,

$\therefore$ the coefficient of increase of pressure $= \dfrac{2{\cdot}78}{760} = \dfrac{1}{274}$.

The above experiment leads us to write

$$P_t = P_0 (1 + c_{ap} t),$$

where $c_{ap} = \frac{1}{274}$, and t is the temperature.

The correction for the expansion of the bulb may be applied in the usual manner. Thus $c_p = c_{ap} + c$, but c is so small that as a rule it is smaller than the probable error in c_{ap}, hence the correction is negligible.

Exp. 26. To prove that the coefficient of increase of pressure is the same for all gases. Take a long piece of glass tubing, bend it as shown in Fig. 47 (which is a perspective view) and suck up some mercury or strong sulphuric acid to act as a pressure gauge in the U tube. Put the ends of the tubes through two rubber corks, which are of such a size as to fit tightly into two flasks, A and B, (which need not be of the same size). By passing a continuous stream of a gas, say coal gas, by means of a tube through the flask A for some time it may be filled with coal gas to the exclusion of any other gas. Similarly fill B with any other gas or let it remain full of air. Place the flasks side by side in a big beaker and fill the beaker with water. Push the corks into the flasks. The levels of the liquid in the two sides of the gauge should now be the same.

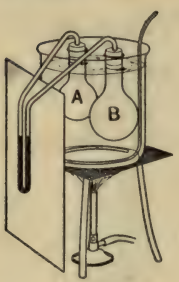


Fig. 47.

Heat the water to boiling, meanwhile inserting a screen between the gauge and the beaker. Observe that *the liquid in the gauge is undisturbed.* But we know by former experiments that the pressures in the two flasks must have increased; we therefore now deduce that they have increased by the same amounts, *i.e.* the coefficient of increase of pressure is the same for coal gas as for air. The same holds for most gases.

Summing up the results of the last two experiments we get the following law:—

The Law of Pressures.—If the temperature of any given mass of gas, whose volume is kept constant, be increased 1° C. the pressure is increased by $\frac{1}{273}$ of the pressure at 0° C.,

i.e. $P_t = P_0 (1 + cp\, t)$ where $c_p = \frac{1}{273}$. .....(4)

Also $$P_{t1} = P_0 (1 + c_p t_1),$$

$$\therefore \frac{P_{t1}}{P_t} = \frac{1 + c_p t_1}{1 + c_p t}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

a useful relation.

58. The relation between pressure and temperature for gases at constant volume was also investigated by Regnault using the apparatus already described in Art. **55** (Fig. 45). The mercury column was always brought back to m in order to keep the volume constant. This was done by pouring mercury in at E. As a result of his experiments Regnault found $c_p = \cdot 003665$, which is slightly less than c_r. Similar results were obtained for other gases. The two coefficients are so nearly equal that for all practical purposes they are taken as identical and equal to $\frac{1}{273}$ (which equals $\cdot 003663$). In the following articles c or $\frac{1}{273}$ will be used indiscriminately for c_r and c_p.

59. The gas thermometer. One of the most important applications of the expansion of gases is the use of gases as thermometric substances. As gases expand much more than either solids or liquids a gas thermometer is much more sensitive than a mercurial one, the expansion of any gas for 1° C. being more than twenty times that of the same volume of mercury. Further, the expansion of a gas is so great compared with that of the vessel which contains it, that it is necessary to know only the approximate law of expansion of the latter.

If we take a mercury thermometer as our standard of temperature there is no option but to say that mercury expands uniformly with rise of temperature, and, strictly speaking, there is no meaning in the phrase. Also using a mercury thermometer to give our scale of temperature it has been found that gases do not expand absolutely uniformly. If, however, a gas thermometer graduated from its own indications be used, of course it may be said the gas expands uniformly with the rise of temperature and also that mercury does not expand uniformly. There is, however, a third scale of temperature, the thermo-dynamic

scale, which is of immense theoretical importance in the study of the higher part of this subject, and therefore more preferable in a way to either the above. It is, however, too difficult to deal with here, and we must content ourselves by saying that since the gas-scale of temperature agrees more closely with the thermo-dynamic scale than the mercury-scale does, the gas-scale is taken as the absolute scale of temperature, and all temperature indicators made for practical use are graduated by comparison with a gas thermometer. It does not greatly matter what gas is used in the thermometer as long as it is a fairly permanent gas, *i.e.* one whose point of condensation is a long way below the ordinary ranges of temperature, and thus we find air, oxygen, hydrogen, and nitrogen being employed by different observers, but perhaps nitrogen, from its chemical inactivity, is the best one to employ. There are other important reasons for the adoption of air, or other gas which does not easily condense, as a thermometric substance; these we shall consider later (Art. **75**), when the question of specific heat has been dealt with.

Theoretically, the air thermometer may take one of two forms; any apparatus, by means of which the mean coefficient of expansion of air at constant pressure, or the mean coefficient of increase of pressure at constant volume has been studied, may be used as an air thermometer; for since c be known we can use the relation, $V_t = V_0 (1 + c\,t)$, to determine t, thus—

$$t = \frac{V_t - V_0}{V_0\, c},$$

or we can use the relation $P_t = P_0 (1 + c\,t)$ to determine t, thus—

$$t = \frac{P_t - P_0}{P_0\, c}.$$

We may therefore use as an air thermometer either of the forms of apparatus described in Arts. **55**, **57**. In practice, however, only one form is used. Although the great expansion of gas is in one way an advantage, it is a great inconvenience if we wish to measure temperature by expansion at constant pressure, for the expansion is

so great that a comparatively large proportion of the gas necessarily occupies, on expansion, portions of the apparatus which are not at the temperature it is desired to measure. For this reason it has been found most convenient to employ the apparatus used to determine the mean coefficient of increase of pressure at constant volume (Fig. 46). In this apparatus the air is maintained at constant volume, and can therefore be kept, throughout the experiment, at the temperature it is required to measure, and which is given by

$$t = \frac{P_t - P_0}{P_0 c} = 273 \frac{P_t - P_0}{P_0}.$$

Here P_t is known from the barometric height (H) at the time of the experiment, and the difference of level (p) of the mercury columns in the two tubes. It is equal to $(H \pm p)$. P_0 is determined once for all by surrounding the bulb with melting ice and determining H and p in the same way as for P_t above. The scale on the instrument may be directly graduated in degrees once P_0 is determined. For, since the pressure increases by $P_0/273$ cms. of mercury for every 1° C. rise in temperature, it is easily seen that the scale of Fig. 46 may be marked off in steps, each of length $P_0/273$ cms., and then labelled the appropriate degrees.

If pressures are measured by mercury and $P_0 = 760$ mm., the degrees are $\frac{760}{273}$, *i.e.* 2·78 mm. in length.

Owing to the fact that the properties of air remain unchanged through a very wide range of temperature, the air thermometer may be used to determine very high or very low temperatures. When used at very high temperatures the bulb is made of porcelain, and in this form it is a most reliable pyrometer or instrument for measuring high temperatures. The details of construction of air thermometers vary greatly according to the purpose for which they are required; the bulb is often very long and cylindrical, as shown in Fig. 30. For other purposes it is comparatively small and spherical; but in all cases the method of experiment and the details of calculation are identical.

60. Absolute temperature. Imagine an air thermometer constructed out of a long tube of uniform bore in such a way that temperature is indicated by increase of volume at constant pressure, and let the air in the tube occupy 273 cms. of the length of the tube when at 0° C. If this thermometer were plunged into steam produced at the normal atmospheric pressure the air would expand, showing an increase of volume equal to $\frac{100}{273}$ of its volume at 0° C. Hence, at 100° C. the air would occupy 373 cms. of the length of the tube, and if this space of 100 cms., between the freezing point and boiling point, be divided into 100 equal parts of 1 cm. each, then each division will correspond to 1° C. If now this method of division be carried down the tube below the freezing point, the reading at the bottom will be — 273° C. This temperature is called the *absolute zero*. If temperatures be reckoned from this point *as zero*, then the freezing point, which is 273° C. higher, is denoted by 273° Absolute (written as 273° A.), and the boiling point is at 373° A., or, generally, $t°$ C. becomes $(273 + t)$ on the absolute scale, *i.e.* the temperature $t°$ C. $= (273 + t)°$ A., which is usually written T° A. Since this absolute zero is at the *bottom* of the tube it would seem to imply that, at this temperature, the volume of the air is reduced to nothing. This result is, of course, physically impossible, and only presents itself if we falsely assume that air remains a gas during the whole range of temperature between 0° C. and — 273° C.; this we know is not the case, for at a certain temperature it would liquefy,* and at a lower temperature become solid; and consequently its law of contraction would altogether change at these very low temperatures.

In dealing with questions relating to gases it is exceedingly convenient to measure temperatures from this absolute zero, for if we do so we may state that the volume of a given mass of gas at constant pressure is proportional to its absolute temperature. Thus, in the case of the uniform air

* Air liquefies at — 192° C., *i.e.* 81° A. It is therefore perhaps better to define Absolute Temperature as temperature on such a scale that the volume of a given mass of a not easily liquefied gas is proportional to the temperature. On this definition we need not worry about the absolute zero.

thermometer tube mentioned above, if the volume of 1 cm. of its length be denoted by v, then the volume of the air at 273° A. is 273 v, at 373° A. it is 373 v, and at T° Absolute it is given by T v, that is—

$$\frac{V_t}{V_{t'}} = \frac{273 + t}{273 + t'} = \frac{T}{T'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where T and T′ denote absolute temperatures.

This can be derived from formula (3) (Art. 56), viz.

$$\frac{V_t}{V_{t'}} = \frac{1 + ct}{1 + ct'}.$$

Here, $c = \frac{1}{273}$, therefore—

$$\frac{V_t}{V_{t'}} = \frac{1 + \frac{1}{273}t}{1 + \frac{1}{273}t'} = \frac{\frac{273 + t}{273}}{\frac{273 + t'}{273}} = \frac{273 + t}{273 + t'} = \frac{T}{T'}.$$

This enables us to state **Charles'** or **Dalton's Law** in the simple form—

The volume of a given mass of any gas at constant pressure varies directly as the absolute temperature,

$$i.e.\ \frac{V}{T} = \text{a constant when P is constant.}$$

Example. *A given mass of oxygen occupies* 24·0 *c.c. at* 15° C. *Find its volume at* 25° C. *if the pressure is unchanged.*

First change the Centigrade temperatures to absolute temperatures. They are 273 + 15 and 273 + 25. Then apply Dalton's Law, and we get

$$\frac{24}{273 + 15} = \frac{V_{25}}{273 + 25};$$

$$\therefore\ V_{25} = 24 \times \frac{298}{288} = 24{\cdot}8 \text{ c.c.}$$

In the same way if we consider the law of variation of pressure with temperature we shall find, that the pressure exerted by a given mass of gas, at constant volume, is proportional to its absolute temperature. Thus, if the pressure at 0° C. be that due to 273 mm. of mercury, then since the

pressure at constant volume increases or diminishes by $\frac{1}{273}$ of its value at 0° C. for a rise or fall of 1° C. we have—

the pressure at 100° C. $= 273 + \frac{100}{273}.273 = 373$ mm.

,, ,, $- 1°$ C. $= 273 - \frac{1}{273}.273 = 272$ mm.

,, ,, $- 10°$ C. $= 273 - \frac{10}{273}.273 = 263$ mm.

,, ,, $- 273°$ C. $= 273 - \frac{273}{273}.273 = 0$ mm.

If now we re-write this table and express the temperatures as absolute temperatures we have—

pressure at 373 A. T. is that due to 373 mm. of mercury.

,, ,, 272 ,, ,, ,, 272 ,, ,,

,, ,, 263 ,, ,, ,, 263 ,, ,,

,, ,, 0 ,, ,, ,, 0 ,, ,,

This shows that the pressure is directly proportional to the absolute temperature. This result may be deduced directly from formula (5) (Art. 57) for,

$$\frac{P_t}{P_{t'}} = \frac{1 + ct}{1 + ct'} = \frac{1 + \frac{1}{273}t}{1 + \frac{1}{273}t'} = \frac{273 + t}{273 + t'} = \frac{T}{T'} \quad \ldots\ldots\ldots\ldots (7)$$

The Law of Pressures now becomes:

The pressure of a given mass of any gas at constant volume varies directly as the absolute temperature,

i.e. $\frac{P}{T}$ = a constant when V is constant.

The two relations (6) and (7) which we have thus established are of great importance. It must be remembered that (6) is true only if pressure is constant, and (7) is true only if volume remains constant.

Example. A given mass of gas at $- 73°$ C. exerts a pressure of 60 cms. of mercury. What pressure will it exert at 27° C. if the volume is unchanged?

Apply the Law of Pressures and we get

$$\frac{60}{273 - 73} = \frac{P}{273 + 27};$$

$$\therefore P = 60 \times \frac{300}{200} = 90 \text{ cms. of mercury.}$$

61. The general relation between pressure, volume, and temperature for a gas. If P, V, and T denote the pressure, volume, and absolute temperature of a given mass of gas it is possible to establish, for these three quantities a simple general relation which includes the three relations discussed in this chapter.

From Boyle's and Dalton's Laws we know that

$$V \text{ varies as } \frac{1}{P} \text{ when T is constant (Boyle's Law),}$$

and V varies as T when P is constant (Dalton's Law).

Hence, when both T and P vary, we have that

$$V \text{ varies as } \frac{T}{P}.$$

That is,
$$V = R\frac{T}{P}$$

where R is a constant.

This is conveniently written in the form

$$\frac{PV}{T} = R \text{ (a constant)} \quad \ldots\ldots\ldots\ldots (8)$$

That is, however P, V, or T may vary the quantity PV/T for a given mass of a given gas remains constant.

This is a very important formula, and should be remembered.

From the above simple formula it is easy to calculate what volume a given mass of gas will occupy at any temperature and pressure if the volume at any other temperature and pressure is given.

Example. A quantity of gas, measured at 27° C. and a pressure of 740 mm. has a volume of 400 c.c. Find the volume of the gas at 0° C. and 760 mm. pressure.

For 1st state of gas

$$T_1 = 273 + 27 = 300,\ P_1 = 740,\ V_1 = 400.$$

For 2nd state of gas

$$T_2 = 273,\quad P_2 = 760,\quad \text{and } V_2 \text{ is required.}$$

Therefore applying $\frac{PV}{T} = \text{a constant}$

we get
$$\frac{740 \times 400}{300} = \frac{760\ V_2}{273}.$$

Hence
$$V_2 = 354 \text{ c.c.}$$

62. The differential air thermoscope. Leslie's differential thermoscope (Fig. 48) consists of a bent tube terminating in two equal bulbs containing air; the bend of the tube contains coloured sulphuric acid,* and the quantity of air in the bulbs is so adjusted that, when both are at the same temperature, the columns of acid, on each side of the horizontal part of the bend, are at the same height. Thus constructed the instrument is very sensitive to *difference* of temperature of the air in the bulbs; the expansion of the air in the warmer bulb depresses the column of acid nearest it and correspondingly raises the other column.

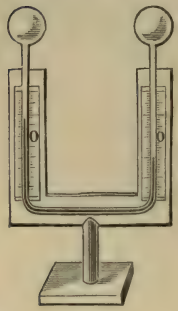


Fig. 48.

The thermoscope is graduated by experiment, but as a rule it is used only to indicate and not to measure changes of temperature.

EXERCISES VI.

1. State Boyle's Law and explain how it may be experimentally proved.

2. State the effect on the volume of a given mass of air of altering its temperature without altering its pressure; also the effect on its pressure of altering its temperature without altering its volume.

3. A cubic metre of gas at 760 mm. pressure is subjected, at constant temperature, to a pressure of 2280 mm. Find its volume.

4. A litre of hydrogen, at 10° C., is heated at constant pressure to 293° C. Find its volume.

5. Air is enclosed in a vessel at 0° C., and, the volume being kept constant, the temperature is lowered to − 88° C., at which temperature the pressure is found to be 385 mm. Find the pressure at 0° C.

---

* Any indicating liquid may be used, but water or spirit are not well adapted because of their volatility, mercury is unsuitable by reason of its great density, and glycerine and oil by reason of their viscosity. Coloured sulphuric acid is therefore commonly used.

6. Find the volume occupied at 0° C. and 760 mm. pressure by 500 c.cms. of oxygen measured at 10° C. and 750 mm.

7. A thousand cubic centimetres of air at 50° C. are cooled down to 10° C., and at the same time the external pressure upon the air is increased from 750 mm. to 765 mm. What is the volume reduced to, the coefficient of expansion of air for 1° C. being 0·00366?

8. In a determination of the coefficient of expansion of air by Gay Lussac's method the volume of the air in the tube was found to be 240 c.cms. at 0° C. and at 77° C. its apparent volume was 310 c.cms. Find the value obtained for the required coefficient. The mean coefficient of cubical expansion of glass is 0·000026.

9. A porcelain air thermometer is used to determine the temperature of a furnace. The excess of the pressure of the air in the bulb over the atmospheric pressure is found to be that due to 1843 mm. of mercury. Find the temperature of the furnace, given that the barometric height at the time of determination equals 758 mm. and that at 0° C. the pressure of the air in the bulb was equal to that of the atmosphere.

10. A volume V of gas at pressure P and temperature T is heated (1) at constant pressure and (2) at constant volume to a temperature T′. Express in terms of P, V, T, and T′ the resulting *state* in each case. [The *state* of a given mass of gas is expressed by giving its pressure, volume, and temperature.]

11. Ten litres of hydrogen at 20·0° C. and 750·0 mm. pressure occupy a volume of 9532·4 c.cms. at 10·0° C. and 760·0 mm. pressure. Find the mean coefficient of expansion of hydrogen.

12. A litre of hydrogen weighs 0·090 gramme at 0° C. and 760 mm. pressure. Find the weight of a litre at 20° C. and 766 mm. pressure.

13. Compare the density of air at 10° C. and 750 mm. pressure with its density at 15° C. and 760 mm. pressure.

14. A flask containing air is corked up at 20° C. and heated in an air bath. A pressure of two atmospheres inside the flask will force the cork out; at what temperature will this take place?

15. At what temperature will a volume of air at 0° C., when heated at constant pressure, double its volume?

16. Find the temperature of absolute zero on the Fahrenheit scale.

17. A quantity of air occupies 10 cubic feet at 0° C., under a pressure of 20 inches of mercury. What will be its volume at 30° C., under a pressure of 1200 inches of mercury?

18. The volume of a certain quantity of air at 50° C. is 500 cubic inches. Assuming that no change of pressure takes place, determine its volume at − 50° C. and 100° C. respectively.

19. 500 cubic centimetres of oxygen gas are measured when the temperature is 20° C., and the temperature is then raised to 40° C., the pressure meanwhile remaining invariable. What is the volume of the oxygen at the latter temperature? (The coefficient of expansion of oxygen is $\frac{1}{3000}$.)

20. A thousand cubic inches of air at a temperature of 30° C. are cooled down to zero, and at the same time the external pressure upon the air is doubled. What is the volume reduced to, the coefficient of expansion of air for 1° C. being 0·00366?

EXAMINATION QUESTIONS.

1. In what different ways does heat modify the properties of bodies?

2. Distinguish between heat and temperature.

3. Define temperature, and show that temperature in the science of heat plays a somewhat similar part to that played by pressure in hydrostatics.

4. What is a thermometer? What should be the characteristic properties of the substance used in a thermometer?

5. Does a thermometer measure temperature or heat?

6. Describe the process of filling and closing and graduating a mercurial thermometer, and give the reason for each separate step.

7. Carefully describe the method of determining that point on the thermometric scale which corresponds to the boiling point of water.

8. In determining the boiling point, which temperature is registered, that of the boiling water or that of the steam it gives off? Why?

9. In graduating a thermometer, which is determined first, the boiling point or the freezing point? Why? Is it sometimes found that when a mercurial thermometer is placed in melting ice, the temperature indicated is not zero, but a fraction of a degree above zero? How is this explained?

10. What other substances besides mercury have been used for the *measurement* of temperature? and what are their respective advantages?

11. Describe two forms of maximum and minimum thermometers.

12. What are the principal sources of error in a mercurial thermometer? and how may they be eliminated?

13. Prove the formula—

$$\frac{F - 32}{9} = \frac{C}{5} = \frac{R}{4}.$$

14. A thermometer of even bore is equally graduated on the stem from $-10°$ C. to $+110°$ C. How would you test the correctness of its indications?

15. Explain precisely what you understand by the *temperature* of a body.

Show that the changes in length of a metal rod on heating could be used to construct a scale of temperature.

Why would water be unsuitable as the fluid in a thermometer intended to register temperatures between 0° C. and 10° C.?

16. Define the coefficients of linear and cubical expansion of a solid. How are they related to one another? Give in detail the experimental method of determining one of them.

17. Define the coefficient of linear expansion of a solid; and describe carefully one method of measuring it. In calculating the expansion of a solid, it is usual to take account of the *rise* only in temperature; but in calculating the expansion of a gas we require to know the initial temperature as well as the rise in temperature. Explain why.

18. Explain how the length expansion of a rod with rise of temperature may be accurately measured.

Explain, with the aid of a carefully drawn figure, how a pendulum may be made of two metals so that its length is nearly independent of variations of temperature.

19. A sheet of plate glass, 100 inches high, is contained in an iron frame which it just fits at 0° C. What will be the width of the gap at the top at 25° C., if the coefficient of expansion of the glass is $8/10^6$ and that of the iron is $12/10^6$?

20. What is meant by the *coefficient of linear expansion* of a solid? Does the number expressing the coefficient depend upon (1) the unit of length, (2) the thermometric scale, employed?

The length of a brass rod appears to be exactly 30 cms. when measured at 20·0° C. by a steel scale correct at 0° C. Find the actual length of the rod at 20·0° C. and at 0° C.

[Coefficients of linear expansion, brass = ·000018, steel = ·000012.]

21. What is meant by the statement that the coefficient of linear expansion of steel is 0·000012? Deduce the value of the coefficient of cubical or volume expansion.

Steel rails 30 feet long at 0° C. are laid at 15° C. Calculate what space must be left between consecutive rails if the highest temperature they attain during the year is 40° C.

22. Explain the construction of the gridiron pendulum.

23. Has a liquid got a coefficient of linear expansion? Describe the method of Dulong and Petit for measuring the coefficient of absolute expansion of mercury.

24. Distinguish carefully between the real and apparent coefficients of expansion of a liquid.

A glass flask which holds 100 grammes of a given liquid at 20° C. holds 98 grammes of the same liquid at 100° C. Find the coefficient of expansion of the liquid, neglecting that of the glass.

If you were given the coefficient of expansion of the liquid how would you find that of the glass?

25. Define the terms coefficient of linear expansion and coefficient of volume expansion. What relation exists between them?

The height of the barometer at 0° C. being 76 cms., what would be the height at the same pressure at 10° C.? [The coefficient of volume expansion of mercury is ·00018.]

26. The graduations of a mercury thermometer between 0° and 100° are equidistant, and the boiling point was marked when the bulb and the whole of the stem were at 100° C. Assuming the coefficient of expansion of glass to be negligible and that of mercury to be ·00018, find what the reading of the thermometer would be if the parts below and above the 0° mark were at 100° C. and at 20° C. respectively.

27. The reading of a barometer at 20° C. is 765·0 millimetres. What would the reading be if both the mercury and scale were 10° C.?

[Coefficient of linear expansion of brass = ·000019.
Coefficient of cubical expansion of mercury = ·00018.]

28. Explain how you would set up an experiment to study the expansion and contraction of water between 0° C. and 30° C. Describe what you would observe at different temperatures, and the conclusions you would draw from your observations.

29. A hollow sealed glass bulb is weighted so that it only just floats in water at 0° C. Discuss what happens to the bulb as the water is gradually raised in temperature.

30. State the Laws of Boyle and Charles.

A barometer tube is partly filled with air at 0° C., and held upright in a wide deep vessel containing mercury. The height of the column of the mercury in the tube is 50 cms., and the barometer pressure is 76 cms. By how much must the vessel be raised or lowered, the tube being fixed, in order that the position of the meniscus in the tube may remain unaltered when the enclosed air is heated to 100° C.?

31. Describe the details of an experiment for determining the increase of pressure of a constant volume of air as the temperature rises from 0° C. to 100° C.

A thin bulb, containing air at 41° C., is sealed up, and placed in an oil bath. The bath is heated gradually until the bulb bursts at 198° C. Find in atmospheres the bursting pressure of the bulb.

32. A copper sphere contains air exerting a pressure of 2 atmospheres at 15° C. Assuming the coefficient of linear expansion of copper to be ·000017, what pressure will the gas exert when the sphere is placed in boiling water?

33. Describe an air thermometer in which the pressure of the enclosed air does not alter with change of temperature. What disadvantage attends the use of such a thermometer?

An air thermometer furnishes a more definite scale of temperature than a mercury thermometer. Are you acquainted with any reason upon which this statement is based?

34. Describe some form of the air thermometer.

35. Explain carefully what is meant by absolute temperature and the absolute zero.

CHAPTER VII.

CALORIMETRY.

63. Quantity of heat. We are already familiar in a general way with the idea of *quantity of heat.* A greater quantity of heat is required to heat a large mass of water from 0° C. to 100° C. than to heat a small mass through the same range of temperature; a considerable quantity of heat is required to convert a pound of ice at 0° C. into water at 0° C.; and a pound of steam in condensing gives out a large quantity of heat. Also a pound of coal in burning gives out a quantity of heat which varies with the quality of the coal; the quantity of heat given out by a gas flame depends upon the quantity of gas used per second, and the completeness of its combustion; and the quantity of heat evolved in a chemical reaction varies with the nature of the reaction and the conditions under which it takes place. The meaning of the term 'quantity of heat' used in this general way is well understood, but it is necessary for a further study of the subject to decide exactly how quantity of heat is to be measured or in what units it is to be expressed.

64. Unit of heat. If quantity of heat is to be *measured* it is evidently essential to decide on a *unit* of measurement. An obviously convenient unit is the quantity of heat required to raise unit mass of some standard substance through one degree. The unit generally adopted is of this type. It may be specified as *the quantity of heat required to raise one gramme of water at* 0° *C. from* 0° *C. to* 1° *C.* Here the gramme is the unit of mass, and the standard substance is water at 0° C. This unit is sometimes

known as the *calorie* or gramme-degree (Centigrade). Other similar units are the pound-degree (Centigrade) and the pound-degree (Fahrenheit), which involve the pound instead of the gramme as the unit of mass.

The unit of heat may be loosely specified as the quantity of heat required to raise one gramme of water through one degree Centigrade of temperature. Unless, however, we specify the temperature from which the water is to be raised, we may not get a constant unit of heat, for it cannot be *assumed*, for example, that the same quantity of heat is required to raise a gramme of water from 10° C. to 11° C. as from 0° to 1° C.

A convenient unit of heat, sometimes used, is the one-hundredth part of the heat required to raise one gramme of water from 0° C. to 100° C. This has been called the *mean calorie*.

It will be shown later that, as heat is a form of energy, it may be measured in energy units such as *ergs* or *foot-pounds*.

Exp. 27. Take 100 grammes of water at 0° C. in a thin glass beaker, and 10 grammes of water at 11° C. in another beaker. Mix the two quantities of water in the first beaker and note the final temperature. It will be found to be approximately 1° C.

Now, in this experiment the 100 grammes of water at 0° C. is raised to 1° C., and therefore gains 100 units. If we assume that there has been no loss or gain of heat in the mixture, this 100 units of heat must have been given out by the 10 grammes of water at 11° C. in cooling to 1° C. That is, 10 grammes of water in cooling through the 10 degrees between 11° C. and 1° C. gives out 100 units of heat. This indicates that each gramme of water in cooling through each degree between 11° C. and 1° C. gives out one unit of heat.

Similarly, if 100 grammes of water at 0° C. be mixed with 5 grammes of water at 21° C., the final temperature will be found by *direct experiment* to be 1° C., thus proving that the 5 grammes of water in cooling through the 20 degrees from 21° C. to 1° C. gives out 100 units of heat, that is, each gramme of water in cooling through any one of the degrees between 1° C. and 21° C. gives out one *unit* of heat.

Further, if any two quantities of water at any temperatures between 1° C. and 100° C. be mixed, the final temperature, *given by experiment*, is always found to be in accord

with the assumption that one gramme of water in cooling through *any* one degree gives out one unit of heat. Hence it is usually assumed that one gramme of water in heating or cooling through one degree of temperature gains or loses one unit of heat wherever the degree be taken between 0° C. and 100° C.

Example. Fifty grammes of water at 10° C. are mixed with 100 grammes of water at 40° C. Find the temperature of the mixture.

Let t° C. denote the final temperature of the mixture.

Then the warm water cools from 40° C. to t° C., and loses $100\,(40-t)$ units of heat.

Also the colder water is heated from 10° C. to t° C., and gains $50\,(t-10)$ units of heat.

Now, if there be no loss or gain of heat for the mixture, the heat lost by the warmer water must be exactly equal to that gained by the colder water, and we therefore, in this case, have

$$100\,(40-t) = 50\,(t-10)$$

or

$$150\,t = 4500$$

and

$$t = 30.$$

That is, the final temperature is 30° C.

It must be noted here that in experiments of this kind it is very difficult to avoid serious errors. The transfer of heat is not confined to the two quantities of water. The vessels containing the water take a part in the exchanges of heat, and there may be considerable loss or gain of heat due to radiation. It will be explained later how these errors may be to some extent avoided.

The results of very accurately made experiments go to show that the amount of heat given out by one gramme of water in cooling through one degree slightly decreases as the temperature rises to 40° C., and then slightly increases with further rise of temperature.

For general practical purposes, however, it may be taken that one gramme of water in heating or cooling through any one degree between 0° C. and 100° C. gains or loses one unit of heat.

65. The Calorimeter. A unit of heat having been fixed, it remains to indicate how quantity of heat may be measured in terms of the selected unit. The apparatus used for the purpose of measuring quantity of heat is known as a *Calorimeter*. It consists essentially of a thin metal vessel (usually of copper, brass, or silver) partly filled with water and fitted with a stirrer and accurate thermometer. The vessel must be arranged so as to be protected as far as possible from loss or gain of external heat. Fig. 49 shows a convenient form of the apparatus.

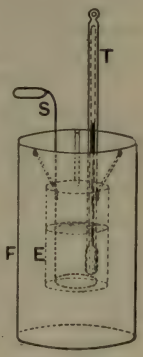


Fig. 49.

The calorimeter E, capable of containing, say, 300 grammes of water, is suspended by three loops of cotton that catch on hooks inside a larger metal vessel F*, which protects it from loss or gain of heat by conduction. The outer surface of E and the inner surface of F are smooth and polished, or plated, to minimise the effect of radiation. The stirrer S is a ring of wire with a vertical branch carrying a small wooden handle. The thermometer T, when in use, should have its bulb well immersed in the water.

To prepare the calorimeter for the measurement of quantity of heat a convenient quantity of water is placed in E, and its mass determined by direct weighing. It is then well stirred and its temperature read just before the hot body is placed into it.

Exp. 28. To measure with a calorimeter the quantity of heat given out by a hot piece of metal dropped into the water in the calorimeter. Prepare the calorimeter as explained above and note the temperature of the water. Drop the piece of hot metal (heated, for example, by immersion in boiling water, or in a steam or air oven) into the water in the calorimeter, and note the final temperature attained when the exchange of heat is complete and the temperature of the mixture is uniform.

---

* Instead of using hooks, the space between the calorimeter and the outer vessel may be packed with *dry* cotton wool or some other bad conductor of heat.

Let, for example, the initial temperature be 10° C. and the final temperature 11·5° C. Then, if the mass of water in the calorimeter is 200 grammes, the quantity of heat given out by the hot metal is that necessary to raise 200 grammes of water through 1·5 degrees of temperature. That is, the quantity of heat given out is $200 \times 1{\cdot}5 = 300$ units. This is evidently the heat given out by the metal in cooling from its initial temperature to 11·5° C.

If the mass of water is known and the change of temperature accompanying any loss or gain of heat be observed, the quantity of heat lost or gained is easily determined. For the unit of heat for ordinary purposes being the quantity of heat required to raise one gramme of water through one degree of temperature, the quantity of heat indicated when 200 grammes of water undergo a change of temperature of three degrees is evidently (200×3) or 600 units. Similarly, if the mass of water in the calorimeter be M grammes and a change of temperature of t degrees be observed, the quantity of heat gained or lost by the calorimeter is Mt units.

Exp. 29. Drop a piece of ice into the water in a calorimeter and determine the quantity of heat absorbed during the melting of the ice. Set up the calorimeter and take the temperature of the water. Drop in the piece of ice and stir carefully until it is completely melted. Note the final temperature when melting is complete and the temperature is uniform.

Let, for example, 20° C. be the initial temperature, 8° C. the final temperature, and 200 grammes the mass of water originally in the calorimeter. Then, as above, the quantity of heat given up by the warm water is (200×12) units or 2400 units. This quantity of heat was evidently absorbed first in converting the piece of ice at 0° C. into water at 0° C., and then in heating this water from 0° C. to 8° C.

66. The "water equivalent" of the calorimeter. In the explanations given in the preceding article the quantity of heat absorbed by the calorimeter, stirrer, and thermometer has been neglected. In Exp. 28, for example, the apparatus absorbs some of the heat given out by the piece of metal, so that the total quantity of heat given out is the 300 units absorbed by the water and a certain unknown number of units absorbed by the apparatus. If the apparatus is of small mass this quantity of heat will be

small, but it cannot usually be neglected. The simplest way of correcting for this absorption of heat by the apparatus is to consider that the presence of the apparatus is *equivalent* to a small addition to the quantity of water in the calorimeter, and to determine the amount of this addition for any particular apparatus by experiment. *The mass to be added to the mass of water in the calorimeter as the equivalent of the apparatus in the absorption or evolution of heat is known as the water equivalent of the calorimeter.*

The experimental determination of the water equivalent for a calorimeter is carried out as follows :—

Exp. 30. Take the calorimeter and fill it about one-third full with water, and note the mass of water taken. Stir till the temperature is uniform. Note this temperature and at once add a mass of water at a different temperature sufficient to about two-thirds fill the calorimeter. Stir until the temperature of the "mixture" is uniform, and note the final temperature. Then determine the mass of water added by finding the gain in the mass of the calorimeter and contents. It is most convenient to take the water in the calorimeter at a temperature of 30° C. or 40° C. and to add to it water *at the temperature of the room.* From the data obtained in an experiment of this kind the water equivalent may be calculated.

Example. For example let the mass of water first taken be 100 grammes and its temperature 35° C. when water at the temperature of the room is added. Let the mass of this added water be 110 grammes and its temperature 15·5° C., and let the final temperature of the "mixture" be 25° C.

Here the heat lost by the warm water first taken is evidently 100 (35-25) units = 1000 units.

Similarly the heat gained by the cold water added is 110 (25-15·5) units = 1045 units. Now of this 1045 units only 1000 units comes from the warm water, therefore the extra 45 units must come from the apparatus in cooling with the water in it through the 10° from 35° C. to 25° C. Hence if the water in the calorimeter were increased by 4·5 grammes this mass would, in cooling through the 10 degrees, give out the 45 units. This 4·5 grammes of water would therefore be the *water equivalent* of the calorimeter.

This result is obtained more concisely by an algebraic method. Thus let x denote the water equivalent of the calorimeter, then the heat lost by the warm water and the apparatus in cooling from 35° C. to 25° C. is $(100 + x)$ 10 units. Also the heat gained by the cold water in being heated from 15·5° C. to 25° C. is $110 \times 9{\cdot}5$ units. Hence, if no heat is lost during the experiment we must have

$$(100 + x)\ 10 = 1045.$$

That is $$10\,x = 45 \text{ or } x = 4{\cdot}5.$$

EXERCISES VII.

1. What is the unit of heat?

2. 80 grammes of water at 100° C. are mixed with 50 grammes of water at 12° C. Find the resulting temperature.

3. A piece of iron weighing 50 grammes was heated to 100° C. and dropped into 10 grammes of cold water at 10° C. The resulting temperature was 40° C. Find the heat given out by 1 gramme of iron in cooling 1° C.

4. 40 grammes of ice at 0° C. were dropped into 40 grammes of water at 80° C. When all the ice had melted the temperature of the water had fallen to 0° C. Find the heat absorbed by one gramme of ice in melting.

5. How may the water equivalent of a calorimeter be found?

CHAPTER VIII.

SPECIFIC HEAT.

67. Capacity for heat or thermal capacity. If *equal* masses of different substances take in (or give out) different quantities of heat when heated (or cooled) through the *same* range of temperature the substances are said to have different *capacities for heat* or different *thermal capacities*. The question whether different substances have different capacities for heat is one of interest and importance. The following simple experiments indicate the general nature of the answer to this question.

Exp. 31. (*a*) Heat 100 grammes of thin sheet lead (loosely rolled) and 100 grammes of water in the same beaker. Provide two other beakers containing equal quantities of water at the ordinary temperature, say 100 grammes at 15° C. in each. Put the hot lead into one of these beakers and the hot water into the other, and observe in each case the rise of temperature produced. The rise of temperature due to the hot water will be found to be much greater than that due to the hot lead. For example, if the water and lead were originally at 100° C. the final temperature in the beaker to which the water is added would be found to be about 58° C., and in the beaker to which the lead is added it would be found to be only about 18° C. That is, 100 grammes of water in cooling from 100° C. to 58° C., *i.e.* through 42°, give out about *fifteen* times the quantity of heat given out by the same mass of lead in cooling from 100° C. to 18° C., *i.e.* through 82°, or, roughly, twice 42°. Hence a gramme of lead in cooling through one degree gives out about *one-thirtieth* of the heat given out by a gramme of water in cooling through one degree. That is the capacity of lead for heat is about one-thirtieth that of water.

(*b*) Heat 100 grammes of iron wire to 100° C. and "mix" it with 100 grammes of water at 10° C. The final temperature will be found to be about 19° C. This indicates that 100 grammes of iron in cooling through 81 degrees gives out the same quantity of heat as is taken in by 100 grammes of water in heating through 9 degrees. That is, one gramme of iron in cooling through one degree gives out about *one-ninth* of the heat given out by one gramme of water in cooling through one degree.

(*c*) Mix 100 grammes of water at 100° C. with 100 grammes of iron at 10° C. The final temperature will be found to be about 91° C. That is the water in cooling through 9 degrees gives out the same amount of heat as the iron absorbs in heating through 81 degrees. This indicates as before that for the same change of temperature the iron gives out one-ninth the quantity of heat given out by the same mass of water. That is the *capacity* of iron for heat is one-ninth that of water.

It should be noticed that when *equal* masses of different substances at different temperatures are "mixed," the changes of temperature produced are inversely proportional to the capacities *of the* substances for heat. Thus, in Experiment 31 *b* and *c*, the changes of temperature for the iron is nine times that for the water, indicating that the capacity of iron for heat is *one-ninth* that of water.

Exp. 32. (*a*) Mix 100 grammes of mercury at 100° with 100 grammes of water at 7° C. The final temperature of the "mixture" will be found to be about 10° C. Here the changes of temperature in the water and mercury respectively are in the ratio 3 : 90 or 1 : 30, and therefore the capacity of mercury for heat is one-thirtieth that of water.

(*b*) Mix 100 grammes of water at 100° C. with 100 grammes of turpentine at 18° C. The final temperature will be found to be about 74° C., showing that the capacity of turpentine for heat is about 24/56 or 3/7 that of water.

When the capacity for heat of a *substance* is spoken of the quantity of heat required to raise *unit mass* of the substance through one degree is usually meant. The capacity for heat of any *body*, such as a piece of apparatus, is the quantity of heat required to raise the whole mass of the body through one degree rise of temperature.

Instead of the term *capacity for heat* the more convenient term *thermal capacity* is often used.

68. Specific heat. The specific heat of any substance may be defined as the number of units of heat (*i.e.* calories) required to raise unit mass of the substance through one degree. That is, the measure of the thermal capacity of unit mass of any substance is the specific heat of that substance. The *mean specific heat* of any substance for any range of temperature is the average value, for that range, of the number of units of heat required to raise unit mass of the substance through one degree.

Thus, if Q units of heat are required to raise unit mass of a substance from $t°$ C. to $t'°$ C. then $Q/(t'-t)$ is the *mean specific heat* of the substance between $t'°$ C. and $t°$ C. Also if s denote the specific heat of any substance the quantity of heat required to raise m units of mass of the substance from $t°$ C. to $t'°$ C. is $m\,s\,(t'-t)$ units.

The *specific heat of a substance at any temperature* is the mean specific heat of the substance for a very short range of temperature including the given temperature. For example, the specific heat of copper at 50° C. may be taken as the mean specific heat between 49° C. and 51° C. It is evident that the shorter the range taken the more exactly does the mean specific heat for the range give the specific heat at a definite temperature.

Example. 100 *grammes of brass in cooling from* 100° C. *to* 20° C. *gives out* 720 *units of heat. Find the mean specific heat of brass between* 20° C. *and* 100° C.

The average quantity of heat given out by one gramme of brass in cooling through one degree is evidently given by

$$\frac{720}{100 \times 80} \text{ units} = \cdot 09 \text{ units.}$$

That is, the mean specific heat of brass between 20° C. and 100° C. is ·09

Example. *Ten units of heat are required to raise* 50 *grammes of copper through* 15° C. *to* 17° C. *Find the mean specific heat of copper between* 15° C. *and* 17° C.

Here the average number of units of heat required to raise one gramme of copper through one degree is

$$\frac{10}{50 \times 2} = \cdot 1.$$

This may also be taken as the rough value of the specific heat of copper at 16° C.

69. Experimental determination of the specific heat of a solid. A very rough determination of the specific heat of a solid may be made by the method of the following experiment.

Exp. 33. Take a piece of brass tube about 100 grammes in mass and weigh it. Attach a piece of thread to it, and immerse it in boiling water so that it can be lifted out when required by means of the thread. Set up the calorimeter as described in Art. 65, and note the temperature of the water in it. Then remove the brass from the boiling water, shake off the adhering water, and transfer it as rapidly as possible into the water in the calorimeter. Now stir the mixture and note the maximum temperature reached.

Example. As an example, let the mass of the brass tube be 95·4 grammes, the initial temperature of the calorimeter 14° C., and the final temperature 18° C. Then if the mass of water in the calorimeter be 200 grammes and the water equivalent of the calorimeter 4 grammes, we evidently have

The heat lost by the brass in cooling from 100° C. (the temperature of the boiling water) to 18° C. is $95{\cdot}4 \times 82 \times 5$ units, where s denotes the mean specific heat of brass between 18° C. and 100° C.

Also, the heat gained by the water and calorimeter is 204×4 units.

Hence, if there is no loss of heat, we have

The heat lost by the brass = the heat gained by the water and calorimeter.

That is,

$$95{\cdot}4 \times 82 \times s = 816,$$

or

$$s = \frac{816}{95{\cdot}4 \times 82} = {\cdot}10.$$

That is, by this experiment the mean specific heat of brass between 18° C. and 100° C. is ·10.

In this experiment the arrangements for heating the brass tube, and for its transfer to the calorimeter, are evidently not satisfactory. The brass may carry water with it, and it may cool several degrees during the time of transfer.

70. Heater for specific heat work. From what has been said it will be clear that for accurate determinations of specific heat a good form of heater is necessary. The following more or less simple arrangements have been found useful for laboratory work. A tin can (6″ long, 4″ diameter) is provided with a loose lid B, pierced by a large central hole and two or three smaller ones ($\frac{1}{4}$″ diameter)

near the edge. A tin tube C (5″ long, $1\frac{1}{2}$″ diameter) is provided with a flange D, which when the tube hangs within the can rests on the lid B.

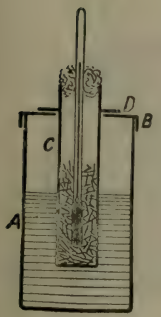


Fig. 50.

Fig. 51.

The substance to be heated should be broken into small pieces.* A known mass of it is placed in the tube and

* This is not necessary if it is a good conductor.

heated by steam from a small quantity of water kept boiling in the can. Place a thermometer with its bulb well in contact with the substance, and plug the open end of it with a piece of cotton wool. When this temperature is steady for at least five minutes the tube is lifted out by means of a pair of clips, and the substance rapidly transferred without splashing to the water in the calorimeter.

The form of heater in general use, however, is that shown in Fig. 51. This is usually made of copper and covered with thick felt. It consists of two concentric

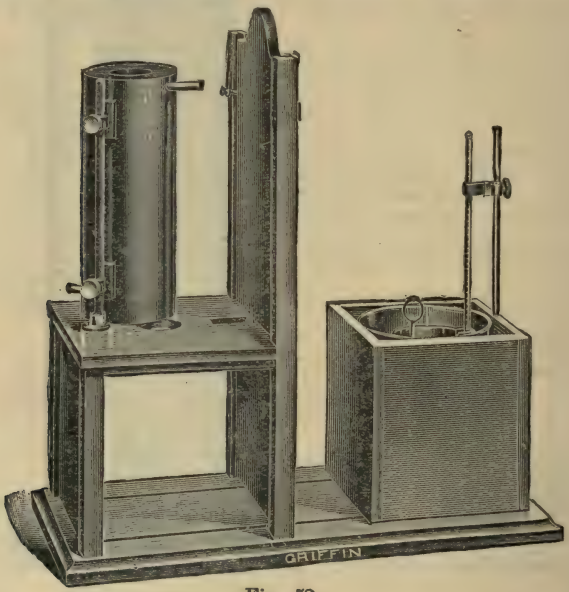


Fig. 52.

cylinders. A current of steam from a boiler is passed through the space between the cylinders. The interior of the inner cylinder is thus a steam oven and objects suspended in it can be heated to nearly 100° C. The method of using this heater is sufficiently indicated in the figure.

Fig. 52 shows the form usually known as Regnault's calorimeter. The calorimeter in its shielding box slides along grooves cut in the base of the frame-work. While the body is heating in the steam-heater a sliding shutter prevents transference of heat from the heater to the calorimeter. When the body has reached the temperature of the heater this sliding shutter is raised and the calorimeter is pushed below the heater. The body is then lowered into the water, the calorimeter quickly slid back into its original position, and the shutter lowered. The water is then briskly stirred and its maximum temperature observed.

71. Specific heat of a liquid. The method of mixtures as described above for a solid may also be used to determine the specific heat of a liquid. Thus, if the liquid be placed in the calorimeter instead of water and a solid of *known* specific heat be heated and "mixed" with the liquid in the ordinary way, the specific heat of the liquid may be calculated from the data of the experiment.

Thus, let 100 grammes of copper (specific heat ·095) be heated to 100° C. and dropped into 200 grammes of turpentine in the calorimeter (water equivalent 4 grammes) at 15° C., and let the final temperature of the mixture be 23·0° C. Then, if s denote the specific heat of the turpentine, we have

$$200 \times 8{\cdot}0 \times s + 4 \times 8{\cdot}0 = 100 \times {\cdot}095 \times 77,$$

$$1600\, s = 732 - 32 = 700,$$

or

$$s = \tfrac{700}{1600} = {\cdot}43.$$

That is, the mean specific heat of turpentine between 15° C. and 23·5° C. is nearly ·43. This is also, roughly, the specific heat of turpentine at about 20° C.

Another method of finding the specific heat of a liquid by the method of mixtures is to enclose the liquid in a small thin flask or tube (made preferably of a good conductor), and experiment with it as in the case of a solid, making proper correction for the thermal capacity of the flask or tube.

Or, if the liquid is cheap and no chemical action occurs between it and water, it may be heated in a beaker and poured directly into the water.

72. The cooling correction in calorimetric experiments. It will have been noticed that in all calculations from the data of a calorimetric experiment it has been assumed that there is no loss or gain of heat by the calorimeter to or from its surroundings. In all actual experiments, however, the calorimeter is subject to a small loss or gain of this kind. As already explained, the construction of the calorimeter is designed to reduce this loss or gain of heat to a minimum. But in all experiments it is desirable to make a correction for this source of error. In most experiments the calorimeter is hotter than its surroundings, and the error is due to cooling during the experiment. For this reason the correction is sometimes called the cooling correction, but the principle of correction is the same whether it has to be applied on account of the loss or gain of heat during the experiment.

One very simple way of making this correction is to arrange the experiment so that the average temperature of the calorimeter during the experiment is the same as the temperature of its surroundings. When this is done the loss of heat while the calorimeter is hotter than its surroundings roughly compensates for the gain of heat when it is colder than its surroundings. It is obvious, however, that to arrange the experiment in this way it may be necessary to make one or two trial experiments.

Example. Suppose that in a specific heat determination the temperature of the calorimeter surroundings is 14° C., and that in a trial experiment a rise of 4 degrees is obtained in the calorimeter. Then, in order to adjust for this method of correction, the initial temperature of the water in the calorimeter should be adjusted to 12° C. The highest temperature reached will then be about 16° C., and the mean temperature during the experiment about 14° C.*

---

* Even this is not quite correct, for on "mixing" the rise of temperature with time is not uniform, but rises more quickly during the first half of the rise than during the second half. The method is therefore only approximate. A better method is to read the temperature of the water every quarter minute after the hot body is put into it until the water has passed its maximum temperature and is cooling. From the curve obtained a fairly accurate correction may be applied. Full details are, however, beyond an elementary book.

73. Determination of specific heat by the method of cooling. This method is occasionally employed for the determination of the specific heat of liquids. If a given mass of liquid, at a known temperature, be placed in a vessel, and allowed to cool in an enclosure (or room) at a lower temperature than its own, then, it is found, by experiment, that the rate of loss of heat depends only upon—

(1) The temperature of the liquid.

(2) The lower temperature of the enclosure (or room).

(3) The nature and extent of the surface at which cooling takes place.

The heat lost is therefore independent of the nature of the liquid, so that if different liquids are allowed to cool under exactly the same conditions (as defined in (1), (2), and (3) above) they will lose heat at the same *rate.** This is the principle applied in the method of cooling.

To determine, by this method, the specific heat of a given liquid, a known mass at a temperature of about 60° C. is placed in a calorimeter provided with a thermometer. The calorimeter and the vessel surrounding it are covered by a lid with a small hole in it for the thermometer stem, and, in order to maintain the surrounding temperature constant, the apparatus is placed in a bath of water at the temperature of the room. The cooling, which now goes on slowly, is carefully watched, and the time of cooling through a given range of temperature, say, from 50° C. to 40° C., is noted. The liquid is then removed from the calorimeter and replaced by an equal volume of water, of known mass, raised to about 60° C. The experiment is repeated, and the time of cooling through the same range (50° C. to 40° C.) is noted. We have now sufficient data for calculating the specific heat of the liquid. Let m be the mass of the liquid, and t its time of cooling from 50° C. to 40° C., m' the mass of the water, and t' its time of cooling from 50° C. to 40° C., also let w denote the water equivalent of the calorimeter, etc

* The rate of *fall of temperature* will, of course, be different in each case.

Then, for heat lost during cooling of liquid we have, if s denote the specific head of the liquid—

Heat lost by liquid during time $t = m\,s \times 10$

,, ,, ,, calorimeter during time $t = w \times 10$.

$\therefore$ Total loss $= (m\,s + w)10$,

and *rate* of loss $= \dfrac{(m\,s + w)\,10}{t}$.

Similarly, rate of loss of heat by water $=$

$$\frac{(m' + w)10}{t'}.$$

But, by the experimental laws of cooling we have—

$$\frac{(m\,s + w)10}{t} = \frac{(m' + w)10}{t'},$$

a relation from which we can calcalute s.

It is to be noted that the specific heat here determined is the mean specific heat for a particular range of temperature (50° C. to 40° C. in this case) and is relative to the mean specific heat of water for the same range of temperature. If the range of temperature is short, the method practically gives the specific heat at the average temperature of the range, relative to the specific heat of water at the same temperature.

Exp. 34. Find the specific heat of petroleum oil by the method of cooling. As a calorimeter use a copper or iron canister coated with lamp-black on its outer surface. Cut two holes in the lid for the thermometer and stirring rod. Support on a cardboard cross (Fig. 53) inside a box of cardboard or wood of about 1 ft. each way, or a large metal canister whose inner surface is coated with lamp-black. This box prevents draughts, facilitates cooling, and supplies an enclosure of almost constant temperature.

Fig. 53.

Weigh the calorimeter, two-thirds fill it with hot water, insert the thermometer and stirrer, place within the box, and observe the time of cooling through 10° C. (say from 60° to 50°, 50° to 40°, 40° to 30°). Then remove and weigh, thus getting m. Heat a sufficient quantity of petroleum in a large test-tube immersed in hot water. Avoid a flame. Fill the calorimeter up to the same level as before with the hot petroleum. Note the times of cooling for the same ranges of temperature. When cold find the mass of the

petroleum. Compare the results for the same ranges of temperature, and so calculate the specific heat of petroleum.

Example. *Determine the specific heat of petroleum from the following experimental results :—*

Mass of calorimeter (of sp. ht. ·1) = 44·6 gms., mass of water = 124·1 gms., mass of petroleum 95·6 gms. Time of cooling from 55° to 50° was 410 secs. for water and 195 secs. for petroleum.

But thermal capacities are proportional to times of cooling thus :

$$\frac{(124{\cdot}1 + 4{\cdot}5)}{410} = \frac{(95{\cdot}6s + 4{\cdot}5)}{195};$$

$$\therefore \quad 128{\cdot}9 \times 195 = 410 \times 95{\cdot}6s + 410 \times 4{\cdot}5,$$

whence $s = {\cdot}59.$

Several other methods of determining specific heat, which depend upon a knowledge of latent heat, will be considered in the next chapter.

74. Specific heat of gases. Regnault made careful determinations of the specific heat of various gases. The gas was collected into a reservoir, and then heated to a high temperature by passing it through a long spiral tube immersed in an oil bath. It then passed through a calorimeter where it was cooled to the temperature of the surrounding water, and then escaped into the air. The rise of temperature in the calorimeter was noted, and the mass of gas which had passed through was determined by the change of pressure in the reservoir. On equating the loss of heat of the gas to the gain of heat of the calorimeter and the contained water the specific heat may be found as in the " method of mixtures " before. Since the temperature of the calorimeter gradually rises during the experiment, the first traces of gas experience a larger fall in temperature than the last traces, and hence when calculating the loss of heat by the gas, the average fall in temperature must be multiplied by the mass of the gas. Thus, for example, if the temperature of the oil-bath is 200° C., and the original and final temperatures of the calorimeter are 10° and 20° the average fall in temperature is

$$\left(200 - \frac{10 + 20}{2}\right) \text{ or } 185° \text{ C.}$$

Regnault's experiments established the following results:—

1. The specific heat of a gas at constant pressure is the same whatever that constant pressure may be.

2. All simple gases have, at the same pressure, the same thermal capacity per unit volume.

3. The specific heat of a gas is the same at all temperatures.

It should be noticed that gases have two specific heats, according as they are heated at constant pressure and allowed to expand (specific heat at constant pressure) or at constant volume without expansion (specific heat at constant volume). The specific heat at constant pressure is greater than the specific heat at constant volume, because, not only has the temperature of the gas to be raised, but a quantity of heat, equivalent to the work done during the expansion against the external pressure, has to be supplied to the gas.

75. Properties of a typical thermometric substance. We are now in a position to consider this question more fully than in Chap. II. Assuming that we wish to indicate change of temperature by change of volume, the chief properties necessary to constitute the substance a typical one are:—

1. *Uniform expansion with increase of temperature.**

2. *A large coefficient of expansion;* that is, the increase of volume for 1° rise of temperature should be sufficiently large to be capable of accurate measurement.

3. *Low thermal capacity per unit volume.* When this is the case—together with (2)—the thermometer is very sensible to loss or gain of heat.

4. *Constant specific heat at all temperatures.* A substance possessing this property would lose or gain equal

* This implies the existence of an absolute scale of temperature that does not depend upon the properties of any thermometric substance.

quantities of heat for each degree change of temperature. For most substances the specific heat increases with the temperature.

5. *The range of temperature, within which the above properties remain unchanged, should be as wide as possible.*

6. *Its power of transmission of heat, either by conduction or convection* (see Chs. XIV., XV.) *should be great.* This insures rapid and uniform distribution of temperature throughout its substance.

The only substance possessing the above properties in the required degree is air, or one of the gases which closely follow Boyle's law, and, for this reason, the air thermometer is the most perfect instrument of its kind. The properties of mercury, between 0° C. and 100° C., very closely conform to those given above.

When a gas is used 3 and 4 apply much more to the bulb of the thermometer than to the gas inside, for the bulb has much the larger capacity of heat.

TABLE OF SPECIFIC HEATS.

Solids.		Liquids.	
Lead	·031	Mercury	·033
Tin	·055	Turpentine	·426
Brass	·094	Glycerine	·555
Copper	·095	Alcohol	·620
Zinc	·096		
Iron	·114	**Gases** (at constant pressure).	
Thermometer Glass	·198	Air	·237
Sulphur	·180	Steam	·481
Aluminium	·214	Hydrogen	3·409
Ice	·481		

76. Effects of the high specific heat of water. Owing to its great specific heat, water absorbs much more heat in warming, and gives out much more heat in cooling, than an equal weight of almost any other substance. Hence the value of water in hot-water pipes, foot-warmers, etc.

Since land is more easily heated or cooled than water, land absorbs more heat from the sun in a given time than water; and, on the other hand, gives up more heat in a given time than water when the sun is down. Now, the

prevailing temperature of an island is between that which the land would have if the water were absent and that which the water would have if the land were absent. Thus in summer, when the sun is above the horizon for a considerable time, the temperature of an island is lower than that of the continent; and in the winter, when the sun is above the horizon for a short time only, the temperature of an island is higher than that of the continent, *i.e.* the climate of an island is much more equable than that of continental countries in the same latitude.

77. The total quantity of heat possessed by a body. It should be noticed that in all calorimetric experiments we have to deal only with the quantities of heat which a body *gains* or *loses*, not with the total quantity of heat which a body possesses. For example, 100 grammes of copper in cooling from 100° C. to 20° C. *lose* about 80 units of heat, or in heating from 20° C. to 100° C. *gain* 80 units, but the total quantity of heat possessed by 100 grammes of copper at, say, 20° C. is a quantity which cannot be determined, for we do not know at what temperature copper may be said to have lost all its heat, neither do we know the average value of the specific heat of copper down to that temperature.

EXERCISES VIII.

Practical

1. Find the specific heat of bronze by the method of mixtures. Take 4 or 5 halfpennies, bore holes through them near their edge, and tie together by string. Heat as explained. They can be used as their own stirrer.

2. Find the specific heat of the following solids by the method of mixtures :—iron nails (150 gms.), brass nails (150 gms.), lead shot (300 gms.), pieces of glass rod and tube (100 gms.), etc.*

3. Find the specific heat of the following liquids by the method of mixtures :*—Put the *liquid* in the calorimeter *instead of* water; then perform the experiment as above. Use methylated spirit, petroleum, alcohol, glycerine, etc., and a solid like iron (nails).

---

* Several determinations of the specific heat of each substance should be made until there is approximate agreement in the results obtained.

4. Find, by the method of cooling, the specific heat of petroleum, alcohol, glycerine, castor oil, aniline, etc.

Calculation.

5. Why should a pound of iron, heated to 100° C., sink further into ice than a pound of lead of the same temperature?

6. Describe an experiment to compare the specific heats of copper and lead.

7. What is meant by *unit of heat*? If the specific heat of iron be $\frac{1}{9}$, and 1 lb. of iron be cooled down from the temperature of boiling water to the temperature of melting ice, how many units of heat are evolved?

8. If the sp. ht. of iron be $\frac{1}{9}$, and 5 lbs. of iron be cooled from 100° to 0° C., how many heat units are lost?

9. What is meant by saying that the specific heat of water is 30 times as great as that of mercury? If a pound of boiling water is mixed with a pound of ice-cold mercury, what will be the temperature of the mixture?

10. If a pound of boiling water is mixed with 3 lbs. of ice-cold mercury, what will be the temperature of the mixture?

11. Ten grammes of water at 98·0° C. are poured into a copper vessel weighing 25 grammes and containing 100 grammes of water at 6·0° C. Find the final temperature of the mixture. Specific heat of copper = 0·092.

12. In order to determine the specific heat of silver, a piece of the metal, weighing 21 grammes, is heated to 98° C. and then dropped into a calorimeter containing 100 grammes of water at 10° C. The final temperature of the mixture is 11° C.; find the specific heat of silver. The water equivalent of the calorimetric apparatus is 3·6 grammes.

13. Ten grammes of common salt, at 91° C., having been immersed in 125 grammes of oil of turpentine * at 13° C., the temperature of the mixture was 16° C. Find, from these data, the specific heat of common salt, supposing no loss or gain of heat to have taken place from without and taking the specific heat of oil of turpentine as 0·428.

14. A mass of 200 grammes of copper, whose specific heat is 0·095, is heated to 100·0° C., and placed in 100 grammes of alcohol at 8·0° C., contained in a copper calorimeter, whose mass is 25 grammes, and the temperature rises to 28·5° C. Find the specific heat of the alcohol.

---

* Water could not be used in this case, as common salt is soluble in water. This point should be noticed.

15. The following data were obtained in an experiment for the determination of the water equivalent of a given calorimetric apparatus.

Weight of apparatus	45·623 grammes.
,, ,, ,, + water	224·583 ,,
Initial temperature of apparatus and water	9·0° C.
Temperature of hot water	78·0° C.
Final temperature	13·2° C.
Weight of apparatus after addition of hot water	236·493 grammes.

[*Note.*—These data are given in the order of their determination in an actual experiment.]

16. A ball of platinum, whose mass is 200 grammes, is removed from a furnace and immersed in 150 grammes of water at 0° C. If we suppose the water to gain all the heat which the platinum loses, and if the temperature of the water rises to 30° C., what is the temperature of the furnace? Specific heat of platinum is 0·031.

[This example indicates a method of measuring very high temperatures.]

17. 280 grammes of zinc (specific heat = ·095) are raised to the temperature 97·0° and immersed in 150 grammes of water at 14·0° contained in a copper calorimeter weighing 96 grammes. The specific heat of copper being ·095, what will be the temperature of the mixture supposing that there is no exchange of heat except among the substances mentioned? What is the water equivalent of the calorimeter employed?

18. A copper vessel containing a thermometer is at 12·0° C.; 50 grammes of water at 60·0° are poured in, and the temperature, after stirring, is found to be 50·0°: find the thermal capacity, or water equivalent, of the vessel and thermometer.

19. Determine the specific heat of copper from the following data:

Weight of copper	16·65 grammes.
,, water in calorimeter	49 ,,
Initial temperature of copper	99·5° C.
,, ,, water and calorimeter	12·0° C.
Final ,, mixture	14·5° C.
Water equivalent of calorimeter, etc.	2·1 grammes.

20. Determine the specific heat of alcohol from the following data:

Weight of copper calorimeter	20·4 grammes.
,, ,, ,, + alcohol	70·5 ,,
,, ,, dropped into calorimeter	10·5 ,,
Initial temperature of calorimeter and alcohol	10·0° C.
,, ,, ,, copper	98·0° C.
Final ,, ,, mixture	12·6° C.

21. A piece of platinum, weighing 120 grammes, is taken from a furnace and at once dropped into 100 grammes of water at 10·0° C., contained in a copper vessel, weighing 21 grammes. The final temperature is found to be 37·0° C. : find the temperature of the furnace.

22. 100 grammes of mercury at 250·0° C. are mixed with 80 grammes of mercury at 15·0° C. in a glass vessel weighing 35 grammes. Find the final temperature of the mixture.

23. Regnault found that 100·5 units of heat were required to raise the temperature of unit mass of water from 0° C. to 100° C., and 203·2 units to raise its temperature to 200° C. Find the mean specific heat of water between 0° C. and 100° C., between 100° C. and 200° C., and between 0° C. and 200° C.

24. Twenty grammes of iron at 98·0° C. (specific heat ·119) are immersed in 80 grammes of water at 10·0° C. contained in a copper vessel whose mass is 15 grammes. Find the resulting temperature, the specific heat of copper being ·095.

25. Twenty pound-degrees of heat are communicated to a metal vessel weighing 8 lbs., and containing 10 lbs. of water. If the specific heat of the metal be $\frac{1}{10}$, in what proportion will the heat be divided between the water and the vessel, and what will be their rise of temperature?

26. 100 litres of hydrogen at 0° C. are first heated to 210° and then passed through a calorimeter whose water equivalent is 5 grammes, and which contains 500 grammes of water. Its temperature is initially 10·0° C. and finally $21\frac{3}{4}$° C., and it is estimated that 3·6 calories are lost by cooling during the passage of the gas. Find the specific heat of hydrogen.

27. In a determination of the specific heat of a liquid by the method of cooling, the weight of the calorimeter (copper) = 16·24 grammes; its weight containing the liquid = 27·18 grammes, and containing water = 30·14 grammes. The times of cooling from 60° C. to 55° C. were 140 secs. with liquid and 330 secs. with water. Specific heat of copper is ·095. Find the specific heat of the liquid.

CHAPTER IX.

CHANGE OF STATE.

Liquefaction and Solidification.

78. Liquefaction and Solidification. In Art. **8** attention was called to the fact that most bodies may exist in three states—solid, liquid, and gas—the application or withdrawal of heat being all that was necessary to change a body's state. This chapter deals solely with the change from solid to liquid, and its reverse, the change from liquid to solid. When a solid substance changes, on heating, from the solid state to the liquid state the process of change is known as *liquefaction* or *fusion*. On the other hand, when a liquid substance, on cooling, changes from the liquid state to the solid state the process of change is known as *solidification* or *freezing*.

79. Temperature and change of state. The object of the following experiments is to determine whether or not, during change of physical state, there is any change of temperature.

Exp. 35. Fill a large beaker with pieces of ice and insert a thermometer. The temperature is 0° C. As the ice is heated and stirred so as to secure uniformity of temperature the thermometer shows no change until the *whole* of the ice is melted. That is during the change from a solid—ice—to a liquid—water—there is no change of temperature until the change is complete, and during the change the water and ice are at the same temperature. If this experiment be repeated a number of times it will be found that the change always takes place at the same temperature. The constant temperature at which this change of state, from the solid to the liquid, takes place is known as the *melting point* of the solid or the *freezing point* of the liquid.

Exp. 36. Take some clean paraffin wax in the solid state and heat it in a suitable vessel until it melts. A thermometer should be fixed in the wax so as to record its temperature during the time of heating. The temperature will be observed to rise as the heating progresses *until the wax begins to melt.* After that it will again be found that the temperature remains constant at a fixed point (about 55° C.) until the process of melting or liquefaction is quite complete.

These experiments indicate an important law of liquefaction. The law may be stated as follows:—*During the liquefaction of a substance the temperature remains constant at a fixed temperature, which is different for different substances.* This temperature for any substance is known as the *melting point* of that substance.

If in the above experiments the heating is continued after liquefaction is complete the temperature of the liquid rises in the usual way. Also if the liquid is cooled until the substance assumes the solid state the temperature changes during cooling and solidification will be found to be an exact reversal of the changes observed during heating and liquefaction. That is, the temperature falls until solidification begins, then remains constant at a fixed point until solidification is complete, and then falls in the ordinary way. Also the constant temperature of solidification is observed to be exactly the same as the constant temperature of liquefaction. This temperature for any substance is sometimes called the **freezing point** for that substance.

80. Change of volume during change of state. The object of the following experiments is to determine if the volume of a given mass of a substance changes with change of state.

Exp. 37. Fill a glass bulb with water and seal up the bulb with blowpipe. Expose the bulb to a temperature sufficiently low to freeze the water. It will be found that the bulb or shell is burst open, thus showing that the ice formed is appreciably greater in volume than the water. In this case, therefore, there is an *increase* of volume in changing from the liquid to the solid state.

Exp. 38. Melt some paraffin wax and pour it into a beaker. When it cools the surface will show a considerable depression, indicating that the volume of the solid wax is appreciably less than that of the liquid wax. In this case, then, there is a *decrease* of volume in changing from the liquid to the solid state.

The results of Exps. 36 and 37 suggest that a change in volume accompanies change of state from solid to liquid, but that for some substances it is an increase of volume, while for others it is a decrease of volume.

Exp. 39. Take a piece of ordinary "compo" tubing, fill it with water and close both ends securely. When exposed to a low enough temperature the water freezes and bursts the pipe.

Exp. 40. Nearly fill a 4-oz. flask with broken ice, and add water to fill the interstices of the ice. Mark the water level near the top of the neck by an elastic band. Then melt the ice by gentle heat, and note the fall of the water level as the melting ice contracts.

81. Change of volume in melting. As a general rule, bodies expand in the act of melting and contract on solidification. In the case of phosphorus this expansion on fusion amounts to 3·4 per cent., sulphur 5 per cent., and stearic acid 11 per cent. This accounts for the fact that when any of these substances are cast in glass tubes they contract away from the glass and easily fall out when solid.

But some few substances, such as water, cast iron, bismuth, contract in melting and expand in solidifying.

82. The expansion of water in freezing amounts very nearly to 9 per cent., so that 11 cub. ins. of water at 1° C. become about 12 cub. ins. of ice at the same temperature. Consequently ice floats in pure water with about $\frac{1}{12}$ of its volume above the surface. If the water is confined, it exerts great force by its expansion on freezing, and thus *in the act of freezing* bursts water bottles and pipes. In the case of pipes the damage does not, however, become *apparent* until a thaw melts the plug of ice that has been forced into the rent, and so allows the water to escape. Much damage is done to brick and stone work by similar means. Rain or moisture from the earth penetrates the

bricks, and on freezing forces the particles asunder. Then, after a thaw, the face of the wall shows evident signs of the destruction. On the other hand, the action of frost benefits the farmer by breaking down the rough clods left by the plough into a fine seed bed. Indeed, the origin of the soil from the subjacent rock is largely due to protracted action of this kind.

If a small cast-iron bottle, with sides half an inch or more in thickness, be filled with water and tightly closed by a screw and then placed in a freezing mixture of ice and salt, the bottle will be burst asunder as soon as the water freezes.

If water contracted on freezing, the ice as it formed would sink to the bottom, exposing a fresh layer of water to the cold air; thus a spell of cold weather would convert our rivers, lakes, and pools into blocks of solid ice.

A lump of cast iron floats on the liquid metal just as ice does on water, and an iron casting owes its sharpness to the expansion which takes place in solidifying, and which forces the metal into intimate contact with every part of the mould. A casting in copper or silver is much less sharp, because these metals contract on solidification.

83. The viscous state during fusion. In studying the change from the solid to the liquid state, it is of interest to determine whether the change takes place abruptly or gradually.

Exp. 41. Melt some lead in an iron ladle and note that the change from the solid to the liquid state is apparently quite sharp and abrupt.

Exp. 42. Melt some pitch or sealing wax in a large porcelain crucible and note that the change from solid to liquid is gradual and that the substance passes through an intermediate *viscous* state.

Exp. 43. Heat a piece of glass tubing in the blowpipe flame. The glass quickly softens and assumes a viscous state. If further heated under proper conditions it would melt to a clear mobile liquid.

A number of substances, solid at ordinary temperatures, such as glass, iron, etc., behave in this way when melting. Similarly, a number of substances, liquid at ordinary

temperatures, such as glycerine, acetic acid, and some other organic acids and oils, pass through an intermediate viscous state in passing from the liquid to the solid state. For these substances the melting point, that is, the temperature at which melting takes place, is indefinite, for the process really extends over a range of temperature which varies with the substance. For other substances for which the change of state is sharply defined, the melting point is fixed and definite and characteristic of the substance.

84. Determination of the melting point of a substance. The methods of determining the melting point of a given substance depend on the nature of the substance—for example, the determination of the melting points of mercury and glass would require somewhat different methods, both because the temperatures are widely different and because the nature of the change of state is different in the two cases (Art. 83). We shall describe methods which are applicable where the melting point is definite and lies between 0° C. and 100° C., and for convenience of description we shall assume the substance under treatment is paraffin wax, which melts at about 52° C.

Exp. 44. Melt some wax in an evaporating dish and dip the bulb of the thermometer in. When the indicated temperature is about 60° remove the thermometer and hold the bulb in the air. At first the bulb is covered with a thin transparent layer of melted wax; as it cools a point arrives at which the wax solidifies, giving the bulb a frosted appearance. At once read the temperature. This is the melting point.

Repeat several times, removing the thermometer from the liquid at temperatures nearer and nearer the melting point.

Exp. 45. Extend experiment by placing a beaker of water over a Bunsen burner, and after each solidification stir the water gently with the thermometer and light the burner. Note the temperature at which the film becomes transparent. The mean of the two readings gives the melting point.

The following results were observed in such an experiment :—

Film becomes frosted on cooling.	Film becomes transparent on heating.	Mean.
52·0	52·3	52·2
52·1	52·5	52·3
	Mean	52·2

Exp. 46. Heat some quill tubing in the blowpipe flame and pull it out to form capillary tubing of about ½ mm. diam. and with very thin walls. Break it up into 10 cm. lengths. Melt some wax in a dish and suck up about 5 cms. of wax into the capillary tubes. Now seal the lower ends of the tubes. Attach such a tube by indiarubber bands to the bulb and lower part of the thermometer as in Fig. 54, and mount the thermometer so that the bulb and tube dip into a large beaker of water with the top of the wax just below the surface. Gently heat the water, stirring it all the time. At a certain stage the opaque solid will suddenly change to a transparent liquid ; note the temperature. It is the approximate melting point. Now remove the burner and, keeping the stirrer going, observe the temperature when the wax becomes opaque (*i.e.* solidifies). The mean of the two readings should give a result somewhere near the truth. Repeat the heating and cooling two or three times, the temperatures not being allowed to depart more than 5° from the melting point, and the changes in temperature being more gradual.

Fig. 54.

The following results were obtained in such an experiment :—

Temp. rising.	Temp. falling.	Mean.
53·5	52·3	52·9
53·5	51·5	52·5
		52·7

The results from this method are not very good, because melting and solidifying are sometimes delayed in a very narrow tube.

By using oil in a suitable bath this method may be extended to temperatures much higher than 100° C.

When a substance is melted and allowed to cool the temperature falls gradually until the freezing point is reached. At this point the temperature remains steady until all has solidified. After this the temperature once more falls.

Exp. 47. Place the wax in a small copper calorimeter or copper tube, melt it and introduce a thermometer into the liquid. Then place the tube inside a beaker * to protect it from air currents, and observe the temperature at regular intervals (say every quarter of a minute) as the wax cools. Plot a curve (Fig. 55) in which times are abscissae and temperatures are ordinates. The temperature at which the curve runs most nearly parallel to the axis of time is the

* The apparatus used in the determination of specific heats by the method of cooling is very suitable for this experiment.

melting point. Fig. 55 represents a curve drawn from an actual experiment with paraffin wax. From it the melting point is seen to be 52·5° C. This method is the most reliable one when it can be used. If the substance is not pure there may be more than one melting point and the curve may run nearly parallel to the axis of time at two or more places.

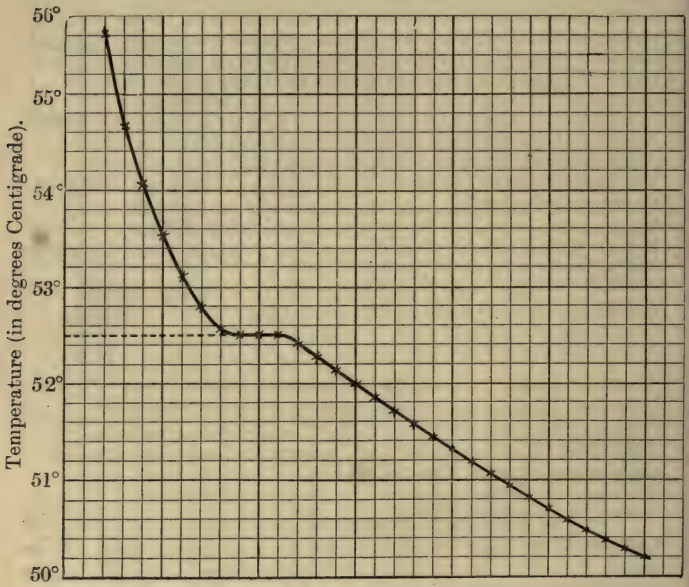


Fig. 55.

85. Melting point of an alloy. By using suitable baths and thermometers the methods of Exps. 46 and 47 may be extended to temperatures much higher than 100° C. Above 300° C. a mercury thermometer may not be used. If Exp. 46 is adopted the air thermometer may be employed, but if Exp. 47 is adopted a quick acting thermometer is required and metallic electrical instruments are then usually employed.

TABLE OF MELTING POINTS.

Platinum	1710° C.	Potassium sulphate	1070° C.
Pure iron	1500° C.	Glass	1100° C.
Copper	1080° C.	Borax	1000° C.
Gold	1064° C.	Sulphur	115° C.
Silver	960° C.	Naphthalene	80° C.
Aluminium	657° C.	White wax	68° C.
Zinc	419° C.	Paraffin wax	52° C.
Lead	327° C.	Phosphorus	44° C.
Bismuth	267° C.	Ice	0° C.
Tin	230° C.	Mercury	−39° C.
Sodium	90° C.	Hydrogen	−253° C.

86. Influence of pressure on the melting point. The influence of pressure on the melting point of a substance may be deduced from theoretical considerations, and was first stated by Professor James Thomson. The nature of this influence is as follows :—

(*a*) If a substance *contracts in melting*, as in the case of ice, increase of pressure causes a *lowering* of the melting point of the substance.

(*b*) If a substance *expands in melting*. as in the case of wax, increase of pressure causes a *rise* of the melting point of the substance,

It has been calculated that, in the case of ice, the melting point is lowered by about ·0073 of a degree Centigrade for one atmosphere increase of pressure.

In the case of ice, which we have seen belongs to the former class of substances, we can illustrate the influence of pressure experimentally.

Exp. 48. Take two pieces of ice and press them together for a few seconds ; on relieving the pressure the two pieces are found frozen together at the region of contact. The pressure enabled a thin layer of the ice to melt by lowering the melting point : on relieving the pressure the water thus formed immediately froze again. This phenomenon is known as **regelation.**

The same process takes place when a handful of snow is squeezed in making a snowball. The outer layer becomes a shell of ice. Again, wheel-ruts over snow are coated with ice : the snow is liquefied as the wheels pass over it,

and the water thus formed freezes when the pressure is removed. The following experiment illustrates the same process :—

Exp. 49. Rest a solid block of ice on two supports and pass a loop of wire round the block between the supports, attach a heavy weight to the loop underneath. The wire gradually cuts its way through the ice, leaving, however, the block quite solid.

Immediately beneath the wire pressure liquefies the ice ; the water thus formed makes its way to a position just above the wire and freezes there.

It is to be noticed that since the ice melting beneath the wire absorbs heat on melting and the water above gives out heat in freezing, the process is helped by having a wire which conducts heat easily ; the heat liberated above is then conducted through the wire to assist in melting the ice below. For instance, it will be found that a copper wire works through more quickly than a steel wire.

Regelation is an important factor in determining the flow of glaciers. At all points where large stresses are set up in the glacier the ice melts, and freezes again as the stress is relieved. In this way stresses in the mass of the glacier are relieved as they arise and the motion of the glacier is facilitated by the process of regelation.

Exp. 50. Melt some crystals of sodium thiosulphate—the "hypo" of the photographer—in a flask. Carefully cork the flask and stand it on one side to cool. The melting point is 45° C., but, if not shaken, it may be cooled to the ordinary atmospheric temperature without solidifying. Now gently remove the cork and drop in a small crystal of the salt. Crystallisation rapidly occurs throughout the mass, and as the mass crystallises the heat rendered latent during the melting of the original crystals reappears as molecular motion and the mass becomes warm to the touch.

87. Latent heat of fusion. It has already been noted that when a substance undergoes liquefaction, or fusion, it absorbs heat, without rise of temperature, during the process. Also when a substance undergoes solidification it gives out heat, without fall of temperature, during the process. The heat thus absorbed or evolved, at constant temperature, during fusion or solidification is called the *latent heat of fusion.* A given substance absorbs, per unit mass, during fusion and evolves, per unit mass, during solidification, a definite quantity of heat which is constant

for the same substance under the same conditions. The latent heat of fusion for any substance may therefore be defined as the quantity of heat required to convert *unit mass of the substance* from the solid state at the melting point to the liquid state at the same temperature. Thus the latent heat of fusion of ice at 0° C. is the quantity of heat required to convert unit mass of ice at 0° C. into water at 0° C. It is also the quantity of heat given out by unit mass of water at 0° C. in becoming unit mass of ice at 0° C. The latent heat of fusion of ice is often called the *latent heat of water*. The heat was supposed to become "latent" in the water during fusion. If the *gramme* be taken as the unit of mass, then the latent heat should be expressed in *gramme*-degrees.

Hence if L denote the latent heat of fusion of any substance then the quantity of heat absorbed during the fusion, or evolved during the solidification of m grammes of the substance is m L units.

Exp. 51. Make a rough determination of the latent heat of the fusion of ice.

Light a Bunsen burner; keep it burning steadily during the experiment. Place a tripod over it and put about a pound of fairly dry ice, pieces about the size of a walnut, into a tin can, also a thermometer (though this is not necessary). Quickly place the can over the flame. Stir gently with the thermometer and note the times

(1) When the can was placed on the flame.

(2) When all the ice has melted (*i.e.* all is water at 0° C.).

(3) When the water commences to boil (*i.e.* when the temperature has just reached 100° C.).

Find the differences in times between (1) and (2), (2) and (3), and note that they are roughly as 4 : 5. Now heat has been flowing into the can at nearly a constant rate, and, seeing that 100 calories are needed to raise 1 gramme of ice-cold water to boiling, the latent heat of fusion of ice is roughly $\frac{4}{5}$ of 100, *i.e.* 80 calories per gramme.

88. Determination of latent heat of fusion of ice. *Principle of the Method.* A quantity of dry ice at 0° C. is placed in a weighed quantity of water warm enough to melt all the ice. The temperature of the water is taken before the ice is put in and also after the ice is melted, finally the weight of ice used is found. From these data the latent

heat of fusion of ice may be calculated. As the experiment lasts some time the cooling correction is of importance, but in this case it may be made negligible by adding pieces of ice until a final temperature is reached as much below the air temperature as the original temperature is above the air temperature.

Exp. 52. Take a calorimeter and shield as in Fig. 49, weigh the calorimeter, two-thirds fill it with warm water at about 30° C., and reweigh. Place the calorimeter in position and accurately read its temperature. Crack up some clear ice to nutmeg size, and after drying each piece carefully with blotting paper drop it into the water. The water becomes colder. Stir well with thermometer. Stop the addition of ice when the temperature is down to about 10° C. or 15° C., and when all the ice has melted stir well, take the temperature, and then take out the thermometer, removing with it as little water as possible. Finally weigh : the difference in the two last weighings gives the mass of ice put into the water. Calculate the latent heat of ice.

Example. *Calculate the latent heat of fusion of ice from the following observations :—*

Calorimeter weighed 52·94 gms., calorimeter and water weighed 189·45 gms., calorimeter and water and ice weighed 215·88 gms.

Initial temperature = 31·0°. Final temperature = 13·8° C.

Air temperature was 22° C. Specific heat of metal calorimeter = ·095.

Heat absorbed by 26·43 gms. of ice in melting = 26·43 L calories.

Heat absorbed by 26·43 gms. of melted ice in warming to 13·8° C. = 26·43 × 13·8 calories.

Heat given out by water and calorimeter in cooling from 31° to 13·8° = (136·51 + 5·03) × 17·2 calories : therefore

$$26{\cdot}43\,\mathrm{L} + 26{\cdot}43 \times 13{\cdot}8 = (136{\cdot}51 + 5{\cdot}03) \times 17{\cdot}2$$

or

$$\mathrm{L} = \frac{141{\cdot}54 \times 17{\cdot}2}{26{\cdot}43} - 13{\cdot}8 = 78 \text{ units.}$$

Note that in this experiment the cooling correction is automatically made negligible.

The latent heat of any other substance could be found in a similar way, *e.g.* by putting some of the solid into some of the liquid or some other hot liquid and noting the fall in temperature. In this case the specific heats of the substance in the solid and liquid states would be required, as well as the specific heat of any other liquid used.

TABLE OF LATENT HEATS OF FUSION.

Ice	80	Silver	21
Nitrate of sodium	63	Tin	14
Nitrate of potassium (nitre)	47	Sulphur	9·4
Paraffin wax	35	Lead	5·5
Zinc	28	Mercury	3

89. The latent heat of fusion of ice at 0° C. is almost exactly 80 units. This value is a comparatively large one, and it is well that it is so. If the latent heat of fusion of ice were small ice and snow on the earth's surface would melt very rapidly with rise of temperature, and disastrous floods would result. In the same way the height of the snow-line would increase, the distribution of glaciers would change, and portions of the earth's surface now covered with snow or ice all the year round would be cleared during a portion of the year. Further, if the latent heat of ice were small our ponds and lakes would freeze very much sooner, and perhaps quickly freeze solid. While this would make more skating available it would be destructive of fish and other life in the ponds and lakes.

90. Solution. When a solid is dissolved in a liquid we have, in general, two distinct effects produced:

(*a*) The physical change of state of the solid.

(*b*) Chemical combination.

In solution, properly so called, only (*a*) takes place, and the process is therefore always accompanied by a fall of temperature due to the absorption of the latent heat of fusion from the solution. For example, the solution of sodium sulphate or ammonium nitrate in water lowers the temperature of the solution to about 15° C., and for this reason these solutions are often used as freezing mixtures. In the majority of cases, however, chemical action takes place as well as simple solution, and we then have two thermal effects corresponding to (*a*) and (*b*) above. The change referred to in (*a*) is always accompanied by absorption of heat, and that of (*b*) always *produces* heat, and thus the final effect depends on whether the absorption of

heat during (*a*) is greater, equal to, or less than the evolution of heat during (*b*). For example, a mixture of 4 parts of sulphuric acid and 1 part of snow produces a *rise* in temperature of 90° C., but a mixture of 1 part of acid and 4 parts of snow produces a fall of temperature of 20° C.

A **freezing mixture** is one in which a low temperature is produced by the absorption of heat resulting from the change of state of some constituent of the mixture. It must be understood that the low temperature of the mixture is only temporary; it is produced *during* the change of state in the mixture, and is complete when that is complete.

The simplest freezing mixtures are those referred to above, where the fall in temperature results from simple solution. A common mixture for laboratory purposes is salt and ice, or salt and snow. In this mixture both constituents change state, and the temperature resulting from this double absorption of heat is − 22° C., which is the freezing point of a definite compound of salt and water.

A mixture of 4 parts of chloride of calcium with 3 of snow produces the low temperature of − 50° C.

91. Ice calorimeters. We have seen that when 1 gramme of ice is melted 80 gramme-degrees of heat are absorbed. This fact has been applied in the construction of ice calorimeters for the determination of specific heat.

Thus, for example, if 50 grammes of copper in cooling from 100° C. to 0° C. melt 6 grammes of ice, then the heat absorbed by the ice is 6×80 units or 480 units, and the heat given out by the copper is $50 \times 100 \times s$ units, where s is the mean specific heat of copper between 0° C. and 100° C. Hence $5000\,s = 480$ or $s = \cdot 096$. Thus the specific heat of the copper is determined.

Hence, in order to calculate the heat absorbed in an ice calorimeter it is necessary to determine with some exactness the mass of ice melted. This may be attempted by two methods: (*a*) by direct weighing of the water produced by the melting of the ice, and (*b*) by measuring the decrease in volume resulting from the change of a portion

of the ice into water. In the second method use has to be made of the fact that the volume of 1 gramme of ice at 0° C. is 1·0908 c.cs., and the volume of 1 gramme of water at 0° C. is 1·0001 c.cs., so that 1 gramme of ice at 0° C. in melting to become water at 0° C. decreases in volume by about ·091 c.c., that is, by about one-twelfth of its volume, or one-eleventh of the volume of the water.

The following experiments illustrate the general principles of the action of ice calorimeters.

Exp. 53. Take a nearly cubical block of ice (A, Fig. 56) about 4 inches edge, and drill a hole about an inch in diameter and an inch deep in it. Make the bottom smooth and slightly conical. Fit over the hole, as a cover, another block of ice, B, about 2 inches thick, as shown in Fig. 56.

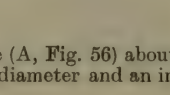


Fig. 56.

Dry the cavity in the block thoroughly with filter paper. [If the cover fits the top of the hole well no heat can reach the cavity from the surroundings of the block. The block may melt *on the outside*, but there is no melting or freezing in the interior unless heat is given out or absorbed by a body placed in the cavity.] Then heat about 50 grammes of, say, brass, and drop it into the cavity. Replace the cover and allow the whole to stand until the brass has time to cool to 0° C. In cooling it gives out heat and a quantity of ice is melted. The water so produced collects in the cavity and remains there *unchanged in quantity* so long as the block is intact. The top of A is slightly rounded so that no surplus water may trickle down the hole. Collect this water by means of a small pipette and a piece of filter paper and determine its mass by direct weighing. The specific heat of brass may be calculated from the observed data by the method of the example given above.

This form of calorimeter was first used by Black.

Example. *Find the specific heat of tin from the following observations:—*

50 grammes of tin were heated to 100° C. and dropped into the ice calorimeter. 3·5 grammes of water were produced.

Arguing as above, we obtain

$$50 \times S \times 100 = 3{\cdot}5 \times 80;$$

$$\therefore \quad S = 28/500 = {\cdot}056.$$

Exp. 54. Take a wide-mouthed 4-oz. bottle, B (Fig. 57), and fit it with a rubber stopper carrying a test tube, T, and a narrow bore ($\frac{1}{2}$ mm.) piece of tubing, t, as shown in figure. Fill the bottle with a large number of small pieces of air-free ice and water and insert the stopper so that all air is excluded. Drop a little clean silver sand into the test-tube, T, and two-thirds fill it with water. Let now the bottle be placed in a larger vessel, V, and completely surrounded with ice, as shown in the figure, so as to protect it from external heating. When the level of the water in t becomes stationary, at a, say, the apparatus is ready for use as a calorimeter. The substance from the heater is dropped direct into the test tube, where it is received on the sand without damage to the tube. The substance then, in cooling to 0° C., gives out heat. This heat melts ice in the bottle and causes a fall in the level of the water in t. If the length of this fall be observed and the cross section of the bore of the tube be known the decrease in volume attending the melting of the ice can be found, and from this the quantity of heat given out by the substance can be calculated. The heat given out by the substance during a known fall of temperature being thus known, the specific heat of the substance can be determined.

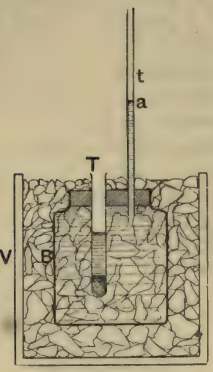


Fig. 57.

Example. *Find the specific heat of brass from the following observations:—*

5 grammes of brass at 100° C. are dropped into the test-tube of above apparatus and the observed fall of level in the indicating tube is 54 mm. The cross section of the bore of the tube is ·010 sq. cm. The decrease in volume due to melting of the ice is $5{\cdot}4 \times {\cdot}01$ c.cm. $= {\cdot}054$ c.cm.

The melting of 1 gramme of ice causes a decrease in volume of ·091 c.cm. Hence a decrease of ·054 c.cm. indicates the melting of $\frac{{\cdot}054}{{\cdot}091}$ gramme or ·59 gramme nearly. The quantity of heat given out by this ice in melting is $·59 \times 80$ units or 47 units. We therefore have

$$5 \times 100 \times s = 47$$

$$\text{or } s = {\cdot}094,$$

where s denotes the specific heat of the brass.

Bunsen's ice calorimeter described below is an improved form of this type of calorimeter.

92. Bunsen's ice calorimeter. The action of this calorimeter depends upon the fact that ice contracts on melting. When 1 gramme of ice melts to 1 gramme of water, the diminution in volume amounts to ·0907 c.cm., and 80 calories of heat are absorbed.

The calorimeter (Fig. 58) consists of a glass vessel, A, having a glass tube, B, fused into it as shown in the figure. At its lower end A passes into the tube tt', which is bent up as shown in the figure, and at t'' is fitted with a cork carrying a bent piece of capillary tubing, c. The horizontal part of this tube is carefully calibrated and graduated so that the volume between any two divisions is accurately known.

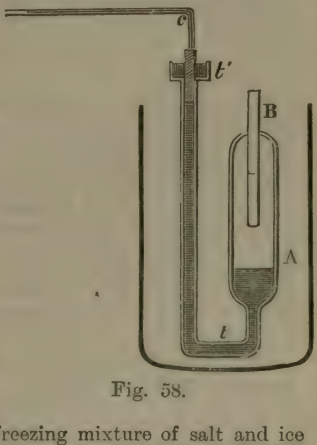


Fig. 58.

To prepare this apparatus for use the upper part of A must be filled with pure, air-free distilled water, and the lower part and the communicating tube with clean, dry mercury. The instrument is then placed in a suitable vessel, *surrounded as completely as possible with pure snow or melting ice*, and a current of alcohol which has been cooled by a freezing mixture of salt and ice is passed through the test tube B until a shell of ice is formed round it in the water in A. The test tube B is now cleaned out, a layer of wool is placed in its bottom, and a small quantity of distilled water added. The instrument is now left, preferably over night, until everything arrives at 0° C. and the position at the end of the mercury column in the horizontal branch of the tube c becomes stationary. Its position is then read.

A known mass of the substance the specific heat of which is required is raised to a known temperature in a suitable heater, and then dropped into the water in B. The heat given out by this substance as it cools to 0° C. melts a quantity of ice in A; a consequent diminution of volume is produced, and *is indicated by the motion of the end of the mercury column* in c towards the calorimeter. When the position of this column becomes stationary a second reading is made, and the difference of the two readings thus taken on c determines the decrease in volume caused by the melting of ice in A. Let the decrease of volume thus determined be denoted, in cubic centimetres, by v. Then, since the melting of 1 gramme of ice in A would produce a diminution of volume equal to 0·0907 c.cm., the mass of ice melted in A must equal $\frac{v}{0{\cdot}0907}$ grammes, and as each

gramme of ice requires 80 calories of heat to melt it, the quantity of heat given out by the substance must equal $\frac{80v}{0{\cdot}0907}$ or $882v$ calories. But, if M denote the mass of this substance in grammes, T the temperature to which it was raised, and S its specific heat, then the heat it gives out in cooling to 0° C. is equal to MST calories. Thus we have

$$882v = \text{MST};$$

$$\therefore \quad \text{S} = \frac{882v}{\text{MT}}.$$

This instrument, when once put in working order, may be used to make several determinations and gives accurate results, but it is rather troublesome to fill and prepare for use. Observe that in this method there can be no error due to exchange of heat between the instrument and the surroundings.

It is evident that, if we work with a substance of known specific heat, we may use this apparatus to determine the latent heat of water or to show that ice contracts on melting, and, by measuring the contraction, to determine the specific gravity of ice. Bunsen made both these determinations, and found 80·0 as the latent heat of fusion of ice and 0·9167 as the specific gravity of ice at 0° C.

93. Nature of latent heat. When a piece of ice at 0° C. is heated its temperature does not rise until all the ice has melted. What has become of the heat given to the ice?

Think of a cannon-ball lying on the ground: it is at rest, and has no destructive power. Think of it now flying through the air at 1,000 ft. a second. It has an immense store of *energy because it is in motion.* Again, think of the ball on the very brink of a precipice, so exactly poised that the least conceivable force would dislodge it and enable it to fall. Again it has a store of energy, but this time *because it is in a particular position*—a position of advantage with regard to the force of gravity, which is ready at any moment to pull it to the ground. The energy is not active at the moment under consideration, but it may become active at any moment. It may be called *potential*, latent, stored energy, or energy of position, to distinguish it from that of the moving ball, which may be called *kinetic* or active or energy of motion.

Now return to the melted ice, and remember that the effect of absorbed heat upon a body is usually twofold. It increases the vibratory motion of its molecules, or, in other words, raises its temperature or increases its molecular *kinetic* energy. But it also drives the molecules further asunder into strained positions, from which they will spontaneously return when the temperature falls. This increases the molecular *potential* energy of the body. In the melting of the ice none of the absorbed heat increases the kinetic energy: it is wholly devoted instead to forcing the molecules into new positions—positions of advantage with respect to some inter-

molecular force, which will drive them back again when the absorbed heat is again lost and the water again becomes solid. We do not know enough about the molecular constitution of water to be able to define those forced changes of position or the force which resists them. It is not simply repulsion in opposition to cohesion; for the ice contracts in melting. It will suffice if we understand that what takes place during the melting is the conversion of the kinetic energy of the absorbed heat into potential energy, and it will enable us to attach a tolerably definite meaning to the term **latent heat**, which is applied to the energy as stored up in the molecules of the liquid water.

When the water again freezes, and the molecules return to their former interrelations, all this latent heat again becomes sensible or kinetic as vibratory motion, and so from the moment the water begins to freeze, until the last drop is frozen, no fall of temperature takes place.

Exp. 55. Cool some water in a test tube by surrounding it by a freezing mixture (Art. **90**). If a suitable temperature indicator be placed in the water it will be found that the temperature falls rapidly at first and afterwards more and more slowly, until finally, if the freezing mixture is effective, a quantity of ice suddenly forms and at the same time a sudden rise of temperature takes place.

The explanation of this is that the water is at first cooled *below* the temperature at which freezing should take place, then, suddenly, freezing takes place, and the heat evolved during the process produces the observed rise of temperature.

EXERCISES IX.

1. If you were to place a pound of lead and a pound of iron, each at 100° C., upon two blocks of ice, which would melt the most ice, and why?

2. Snow is solid water; but I have sometimes suffered from intense thirst on a mountain covered with snow. Why could I not quench my thirst with the snow?

3. Why does a block of ice take so long to melt, even in a warm room? Suppose heat continually poured into it, enough to raise the temperature of a quantity of water equal to the solid part of block 2° C. in a minute. How long would the block take to melt? (Latent heat of water = 80.)

4. What is meant by a *unit of heat*? Taking the specific heat of lead as 0·031 and its latent heat as 5·07, find the amount of heat necessary to raise 15 gm. of lead from a temperature of 115° C. to its melting point, 320° C., and to melt it.

5. If 10 grammes of ice at the freezing point are put into 100 grammes of water at 18° C., what will be the temperature when all the ice has melted?

6. How is it that snow takes so long to melt after a thaw has set in? Describe an experiment to illustrate the principle on which this depends.

7. Explain what is meant by *regelation*, and describe illustrative experiments.

8. Ten grammes of ice at − 10° C. are mixed with 120 grammes of water at 80° C. Find the final temperature of the mixture. (Specific heat of ice = 0·5 and latent heat of water = 80.)

9. How many units of heat would cause a mixture of ice and water to contract by 50 c.mm., if 100 c.mm. of water at 0° C. become 109 c.mm. on freezing.

10. If the specific heat of tin is 0·056, and the latent heat 14·25, what quantity of heat is required to raise 6 lbs. of tin from the temperature 208° C. to its melting point, 238° C., and to melt it?

11. Determine the latent heat of ice from the following data:—

Weight of brass calorimeter (sp. heat ·09) ...	... 35 gms.
,, ,, ,, ,, + water ...	... 156 gms.
Initial temperature of water and calorimeter	... 24° C.
Final ,, ,, ,,	... 17° C.
Weight of calorimeter, etc., after addition of ice	... 165 gms.

12. A gramme of ice at 0° C. contracts 0·091 c.cm. in becoming water at 0° C. A piece of metal weighing 10 grammes is heated to 50° C. and then dropped into a Bunsen's calorimeter. The total contraction is ·063 c.cm.: find the specific heat of the metal, taking the latent heat of ice as 80.

13. Five hundred cubic centimetres of mercury at 56° C. are put into a hollow in a block of ice, and it is found that 159 grammes of ice are liquefied; find the specific heat of mercury.

14. Ten grammes of water at 96° C. are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by 1·09 c.cms.: taking the latent heat of water as 80, what value does this give for the specific gravity of ice?

15. If 100 c.cms. of water in freezing become 109 c.cms. of ice, and the introduction of 20 grammes of mercury at 100° C. into a Bunsen's calorimeter cause the end of the column of mercury to

move through 74 mm. in a tube 1 sq. mm. in section, find the specific heat of mercury. (The heat required to melt 1 gramme of ice is 80 units.)

16. The specific heat of iron is ·113: how many pounds of iron at 260° C. must be introduced into an ice calorimeter in order to melt 2 lbs. of ice?

EXAMINATION QUESTIONS.

1. Define the terms *calorie*, *gramme-degree*, *thermal capacity of unit mass*, *specific heat*, and *water equivalent*.

2. Give the experimental details of the determination of specific heat by the method of mixture.

3. Describe in detail how you would measure the specific heat of copper. State and give the reason for every precaution you would adopt to avoid inaccuracy.

4. What is meant by saying that the specific heat of lead is ·031? 50 grammes of lead shot at a temperature of 100° C. are poured into a beaker containing 30 grammes of turpentine at 15° C. and the temperature of the mixture is found to be 23·4° C. Find the specific heat of turpentine, giving clear reasons for your answer.

5. A body weighing 210 grammes, raised to a temperature of 200° Fahrenheit, is immersed in 570 grammes of water at a temperature of 20° Centigrade. The resultant temperature of both is 28° Centigrade. Calculate the specific heat of the substance. Is there any difference between the specific heat in Fahrenheit and Centigrade units?

6. A copper calorimeter is at 15° C. 30 grammes of water at 30° C. are poured into the calorimeter, and the resulting temperature is 25° C. What is the water-equivalent of the calorimeter?

The calorimeter is then used to find the specific heat of a metal. It contains 65 grammes of water at 15° C. A piece of metal, weighing 123 grammes and at 100° C., is dropped into the water, and the resulting temperature is 25°. Find the specific heat of the metal.

7. Describe how you would measure the specific heat of common salt.

8. Define *specific heat*, and explain what is meant by saying that the specific heat of a substance is variable.

Account for the fact that when 48 grammes of a particular liquid at 20° C. are mixed with 45 grammes of the same liquid at 100° C., the resulting temperature is about 60° C.

Would the resulting temperature be as above if the liquid were water?

9. What meanings can be attached to the term *specific heat of a gas*?

10. How has the specific heat of a gas been determined? What are the principal results obtained from such experiments?

11. A volume of air that would measure 12 litres at normal temperature and pressure is contained in a gasometer. It is then slowly driven through, first, a copper coil heated by a steam bath to 100° C., and then through a thin copper coil in a copper calorimeter containing 300 grammes of water. The calorimeter and its contained coil weigh 200 grammes. During the process the temperature of the water in the calorimeter is observed to rise from 16° to 17° C. Determine from these data and from the following constants the specific heat of air.

Sp. ht. of copper = 0·1; 1 litre of air at normal temperature and pressure weighs 1·294 grammes.

12. State the laws of fusion. What is meant by latent heat of fusion?

13. How do you account for the leakage from water pipes on a thaw after a severe frost?

It is a common practice to leave water taps slightly turned on during a frost to prevent freezing. How do you suppose the freezing is prevented?

14. In order to bring a substance to a definite temperature it is sometimes placed in a vessel containing ice and water. Explain this.

Would the purpose succeed if the vessel originally contained (1) ice only, or (2) much more water than ice?

15. How has the latent heat of water been determined?

16. Describe Bunsen's calorimeter.

17. What is meant by the statement that the latent heat of water is 80?

The temperature of a pond is 8° C. when a freezing wind sets in. Describe what happens as the water parts with its heat, and, assuming that it parts with it at a uniform rate, compare the time taken to the beginning of freezing with the time taken to freeze the top half-inch of water, the total depth being 20 inches.

18. Forty-five grammes of water at 20° C. are mixed with 20 grammes of ice at − 10° C., and, in consequence, half of the ice is melted. The latent heat of fusion being 80 units, find the specific heat of ice.

Describe how you would perform an experiment to test the accuracy of the above numbers, assuming the latent heat of fusion to be correct, and point out the most likely sources of possible error in your experiment.

19. Describe an experiment for determining the specific heat of ice. Explain how the specific heat is found from the data actually obtained in the experiment, assuming the latent heat of fusion to be known.

20. Assuming the specific heat of copper to be 0·1, what quantity of the metal at 100° C. mixed with 100 grammes of water at 15° C. would raise the temperature of the latter to 25° C.?

If the water were contained in a copper vessel weighing 50 grammes, held by strings in an enclosure of temperature 20° C., how would (1) the heat absorbed by the vessel, (2) radiation between the vessel and the enclosure, during the mixing, affect the amount of copper required? Explain.

21. Explain the terms *specific heat*, *latent heat*, and *capacity for heat*.

Given milk at 95° C. and water at 15° C., how much of each would you take, assuming their specific heats to be equal, to obtain a mixture at 40° C. weighing 80 grammes?

Would the final temperature be less if the milk and water were allowed to stand for 10 minutes in a room at 15° C. before mixing than if they were mixed first and then allowed to stand for the same time?

CHAPTER X.

PROPERTIES OF VAPOUR.

94. Change of state from liquid to vapour. When water is boiled steam is given off and disappears. It is evidently distributed through the surrounding air as an invisible gas. Again, if a bottle of volatile liquid such as ether, methylated spirits, or turpentine is exposed to the air, our sense of smell informs us that the substance is diffused in a gaseous state through the air. The process by which substances in the liquid form enter the gaseous state is called *evaporation* or *vaporisation*. It takes place at all temperatures, the only effect of temperature being that of altering the rate at which the process is carried on. Thus, if a saucer of water is placed out of doors, the water will disappear faster in a dry summer than in a dry winter.

95. Vapour pressure. A liquid gives off vapour from its free surface into the adjacent *atmosphere at all temperatures*. Also this vapour is gaseous in character, and may be expected to exert pressure in the same way as a gas does. It is therefore desirable to submit the behaviour of a vapour in this respect to the test of experiment.

Exp. 56. Take a perfectly clean and dry barometer tube about 85 cms. long and from 5 mm. to 1 cm. in diameter and set up a simple barometer with clean, dry mercury.* Note carefully the height of the mercury column by means of a metre scale and compare it with that of a standard barometer. If the reading is more than a mm. below the reading of the standard, the barometer is not satisfactory, and another trial must be made. Now introduce a

* See *Matriculation Hydrostatics*, p. 143.

very small quantity of water into the Torricellian vacuum* of the barometer and note the result. The water may be introduced passing up a very small piece of filter paper soaked in water into the vacuum.

It will be found that the water vaporises in the vacuum almost instantly and that the column of mercury is slightly depressed. For example, the height of the column might be reduced from 758 mm. to 752 mm., the pressure of the vapour thus causing a depression of 6 mm. This evidently indicates that the vapour exerts a pressure which is measured by the depression of the mercury column. Thus, in the example given the pressure of the vapour plus the pressure due to 752 mm. of mercury is equal to a pressure of 758 mm. of mercury, and the pressure of the vapour is therefore equal to that due to 6 mm. of mercury.

Now introduce another very small quantity of water into the barometer tube and again note the result. It will be found that again the water vaporises, but not so quickly as at first, and the column of mercury is again depressed. For example, the height of the column may now fall from 752 mm. to 748 mm., thus showing a further depression of 4 mm.

This indicates that as the quantity of vapour in the enclosed space increases the pressure also increases. In the case of the example given the pressure of the vapour increases from 6 mm. to 10 mm. of mercury.

Repeat the operation of passing a very small quantity of water into the tube several times and note the results.

It will be found that the additional quantities of water vaporise more and more slowly, causing in each case a slight but decreasing depression of the mercury column, until finally the water ceases to vaporise altogether and remains as a thin film on the surface of the mercury, and there is no further depression of the mercury column. This shows that when the space is *saturated* with vapour the pressure of the vapour attains a final value, which is the *maximum vapour pressure* under the existing conditions.

Exp. 57. Repeat the above experiment, using a barometer tube with a large bulb at the closed end. It will be found that a larger quantity of vapour is necessary to saturate the larger space available, but that the maximum vapour pressure is exactly the same if the temperature is the same.

Exp. 58. Repeat **Exp. 56** with alcohol and ether, using a clean tube and clean, dry mercury in each case. It will be found that, at the same temperature, the maximum vapour pressure for alcohol is

---

* This space is not a perfect vacuum, as it contains a very small quantity of mercury vapour derived by evaporation from the liquid mercury. The vapour pressure of mercury is, however, negligible at ordinary temperature. See Art. **99.**

greater than for water and greater for ether than for alcohol. For example, at 20° C. the maximum vapour pressure for water would be about 17 mm., for alcohol about 60 mm., and for ether about 400 mm. of mercury.

Exp. 59. Repeat **Exp. 56** with one of the above liquids, say alcohol, at two different temperatures—for example, in a cold room at 10° C. and in a warm room at 20° C. The maximum vapour pressure will be found to be greater at the higher temperature.

This apparently indicates that the maximum vapour pressure for any substance increases with the temperature.

The experiments described above tend to establish the following facts. A liquid gives off vapour from its free surface at all temperatures, and this vapour behaves like a gas and exerts pressure. Also, when sufficient liquid is present to saturate a closed space with its vapour, the pressure of the vapour attains a maximum value, which is constant for a given liquid at a given temperature, but apparently increases as the temperature rises.

96. Variation of maximum vapour pressure with temperature. The result of **Exp. 59**, described above, indicates that the maximum vapour pressure of a vapour varies with the temperature and suggests that it increases as the temperature rises. It is desirable, therefore, to submit this question of the variation of the maximum vapour pressure with temperature to further experimental investigation.

Fig. 59.

Exp. 60. Set up two simple barometers A, B, as described in **Exp. 56**, and fit them into a cork in a wide jacket tube, J, as shown in Fig. 59. Place a wooden millimetre scale either between the two tubes or behind them and so arrange that readings can be taken to the nearest millimetre.

Fill the jacket tube with warm water, about 60° C., and suspend a thermometer *t* in the water to indicate the temperature. A stirrer *s* should also be arranged to keep the water in constant motion, and so secure uniformity of temperature throughout its mass. A serves as an ordinary barometer. Introduce into the vacuum of B a sufficient quantity of the liquid to be vaporised, say water. As the water in the wide tube slowly cools note the depression of the mercury column in B with respect to A, and so

determine the maximum vapour pressure of water vapour at a number of different temperatures between 60° and the temperature of the room. Then add ice and take readings down to 0° C.

It will be found in studying the result that the maximum vapour pressure increases as the temperature rises and that for *equal* increments of temperature the corresponding increments of pressure rapidly increase. For example, the following are the results of an experiment with water vapour.

Temperature.	Maximum Vapour Pressure in Mms. of Mercury.
60° C.	149
50° C.	92
40° C.	55
30° C.	31
20° C.	17
10° C.	9
0° C.	5

Here for equal increments of temperature, each of ten degrees, between 0° C. and 60° C. the corresponding increments in pressure are 4, 8, 14, 24, 37, and 57 mms. of mercury.

No simple law can be given connecting the increase of pressure with the increase of temperature. When, however, the results of experiment are plotted with temperatures as abscissae and pressures as ordinates a smooth curve of the form shown in Fig. 60 is obtained. It is possible to obtain from this curve by direct measurement the pressure at *any temperature* included in the range of the curve.

Exp. 61. Repeat **Exp. 60**, but begin by carefully filling the outer tube with boiling water. It will be found that the mercury column is depressed almost to the level of the mercury in the cistern, and that the column of vapour is now so long that great difficulty is experienced in maintaining it at a constant temperature. This method of experimenting is therefore not suitable under these conditions. See Art. **104**.

Exp. 62. Repeat **Exp. 60** with (i) alcohol, (ii) carbon bisulphide instead of water, and measure the maximum pressure of vapour for temperatures between, say, 10° C. and 50° C. The general results will be found to be similar to those obtained for water vapour, the pressures at 10°, 20°, 30°, and 40° being respectively 24, 44, 78, 134 mm. of mercury for alcohol, and 200, 300, 435, and 620 mm. of

mercury for carbon bisulphide. We thus see that the maximum pressure of a vapour increases as the temperature rises, and that *equal* increments of temperature show *increasing* increments of pressure.

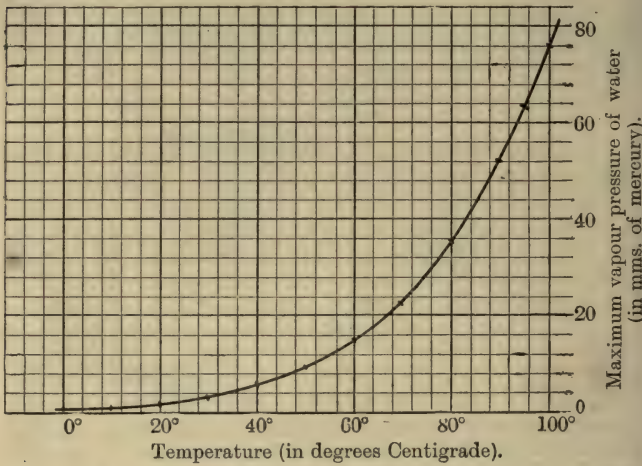


Fig. 60.

97. TABLE OF PRESSURE OF AQUEOUS VAPOUR.

[REGNAULT.]

Temperature.	Pressure.	Temperature.	Pressure.
	mm.		mm.
−10° C.	2·1	70° C.	233·1
− 5° C.	3·1	80° C.	354·6
0° C.	4·6	90° C.	525·5
5° C.	6·5	95° C.	633·8
10° C.	9·2	100° C.	760·0
15° C.	12·7	110° C.	1075·4
20° C.	17·4	120° C.	1491·3
30° C.	31·5	150° C.	3581·0
40° C.	54·9		(=4·7 atmos.)
50° C.	92·0	200° C.	11689
60° C.	148·8		(=15·4 atmos.)

98. Dalton's Laws. So far we have considered the production of vapour in a vacuum, and in the experiments for the determination of maximum vapour pressure the vapour only was present in the closed space. It is necessary now to enquire if the results obtained need to be modified when vaporisation takes place in a closed space containing air or other gas or vapour with which there is no chemical action.

Exp. 63. Find whether the maximum pressure of a particular vapour, say ether vapour, at a given temperature is the same whether vaporisation take place in the presence of a gas, such as air, or in a vacuum. For this purpose the apparatus shown in Fig. 61 may be used. The apparatus is a modification of Boyle's Law apparatus (see Fig. 39). The closed tube V is replaced by the tube A B with two stopcocks S_1 and S_2. Dry air is first introduced into this tube and its pressure measured in the usual way. A small quantity of the liquid to be vaporised is then placed in the funnel of the tube and by opening and closing the stopcocks S_1 and S_2 in order a few drops of the liquid may be introduced into the tube. The increase of pressure due to the vapour may then be determined by arranging the position of the manometer tube, M, so that the air in A B is kept at constant volume. If sufficient liquid is present the maximum pressure of the vapour in the presence of air (or other gas) may thus be determined. It will be found to be the same as in a vacuum.

Fig. 61.

Exp. 64. Show that when two liquids, such as water and carbon bisulphide, which do not react chemically are present in the same space the maximum vapour pressure exerted is the sum of the maximum vapour pressures which would be exerted by the vapours of these liquids if separately confined at the same temperature.

Measure separately by the method of **Exp. 56** the maximum vapour pressures of water and carbon bisulphide at the ordinary temperature and note the values obtained.

Take a piece of barometer tubing, A B C, about 160 cms. long, bend it at the middle to form a U-tube, fill with mercury and invert into a cistern so as to form a double barometer tube of the form shown in

Fig. 62, and pass one liquid, say water, up into one limb, AB, of the tube and the other liquid, carbon bisulphide, into the other limb, CB. The vacuum space will now be saturated with a mixture of the two vapours and the pressure depressing the mercury columns will be the pressure due to this mixture. This pressure should be measured and compared with the sum of the individual maximum pressures of the vapours. It will be found that the pressure of the mixture of the two saturated vapours is equal to the sum of the maximum pressures exerted separately by these vapours.

Fig. 62.

An important point of difference between Exp. 63 and Exp. 64 should be noted. In Exp. 63 the pressure of the air depends, in accordance with Boyle's Law, on the volume it occupies, and in order to get the same *air* pressure before and after the introduction of the vapour it is necessary to adjust to the same volumes in each case. The pressure of a *saturated* vapour at a given temperature is *the same whatever the volume it occupies*, and so no volume adjustment is necessary in Exp. 64.

If other experiments similar to those described above be performed, it will always be found that in the case of a mixture of gases or vapours, or of both, between which there is no chemical action, the pressure of the mixture is the sum of the individual pressures of the constituents of the mixture, each constituent exerting, as a part of the mixture, the same pressure as it would exert if it alone occupied the whole space occupied by the mixture. Thus, if a small quantity of ordinary moist air be passed up into the "vacuum" space of a barometer tube the space becomes occupied by a mixture of nitrogen, oxygen, carbonic acid gas, water vapour, and mercury vapour. Each constituent of this mixture exerts a definite pressure, which is the same as it would exert if it alone occupied the whole space occupied by the mixture, and the pressure, of the mixture is the sum of the pressures that would be exerted by these individual constituents separately.

These results have been formulated as laws which are usually known as **Dalton's Laws**. They may be stated as follows:—

(i) The maximum pressure exerted by a particular vapour, in a closed space, at a given temperature, depends only on that temperature, and is independent of the presence of other vapours or gases having no chemical action on it.

(ii) When several vapours and gases, having no chemical action on one another, are present in the same space, the actual pressure exerted by the mixture is the sum of the pressures which would be exerted by quantities of the constituents, equal to those which are actually present, if separately confined in turn in the same space.

The first of these laws deals with maximum pressure, and is therefore applicable only when the space is saturated with the vapour considered; the second law is applicable whether the space is saturated or not.

In the cases of saturated vapours these laws are only true when these vapours are the vapours of liquids which do not dissolve in or mingle with each other, *e.g.* water and carbon bisulphide. When the liquids mutually dissolve each other, as do water and alcohol, the vapour pressure of the mixture is less than the sum of the vapour pressures which they would separately exert at the same temperature. This could be tested by an experiment exactly similar to **Exp. 64.**

99. The pressure of mercury vapour. Mercury is a liquid and its vapour exerts a pressure which although excessively small at ordinary temperatures is quite appreciable at higher temperatures. Accurate determinations of the maximum vapour pressure of mercury are very difficult to make. The following table gives the latest results:—

Temperature in Degrees Centigrade.	Vapour Pressure of Mercury in Mms. of Mercury.
0	·0002
50	·0122
100	·276
200	17·81
300	248·6
356·7	760

These results explain why in barometer tube experiments the pressure of the mercury vapour necessarily present can be neglected at ordinary temperatures. Other liquids having very small vapour pressures are some oils, glycerine, strong sulphuric acid, etc.

100. Saturated and unsaturated vapours. When a space is saturated with vapour, at a given temperature, this vapour exerts its maximum pressure for that temperature, and is also at its maximum density for that temperature, because the greatest possible quantity of vapour has been introduced into the space. Vapour in this condition is sometimes called **saturated vapour**. When vapour, at a given temperature, exerts a pressure less than its maximum pressure for that temperature, it is said to be **unsaturated.**

Vapour in contact with its liquid, in a closed space, is always saturated, for the presence of the liquid is a proof that the limit of vaporisation, under the existing conditions, has been reached.

The behaviour of saturated vapour under change of pressure and temperature should be considered. **Saturated vapour**, at any temperature, exerts the *maximum* pressure corresponding to the temperature, and hence any attempt to *increase its pressure*, e.g. by compressing it in a closed tube, must necessarily fail; the only effect produced by such an experiment would be the condensation of part of the vapour in the space, because, the vapour being already at its maximum density, for the given temperature, any attempt at further compression causes it to pass gradually into the liquid state, while its pressure remains constant. On the other hand, if saturated vapour be allowed to expand *in the presence of its liquid*, its pressure remains constant, and the liquid present is gradually converted into vapour, because, directly any small increase of volume takes place, the pressure of the vapour, in accordance with Boyle's law, tends to fall below the maximum pressure corresponding to the conditions of the experiment, and hence vaporisation of the liquid present becomes possible, and thus the pressure is maintained constant. If, however, there is no liquid present, the pressure of the vapour decreases on expansion in approximate agreement with Boyle's law, and the vapour becomes unsaturated.

The influence of change of temperature on a saturated vapour also depends on whether its liquid is present or not. If vapour be heated, in a closed space, in contact with its liquid, the liquid is converted into vapour, and the pressure increases at such a rate that, at any given temperature, the vapour present in the space saturates it, and the pressure exerted is the maximum pressure for that temperature. Similarly, if saturated vapour be cooled in a closed space, in contact with its liquid, the vapour condenses and the pressure decreases in such a way that, at any given temperature, the quantity of vapour present in the space is just sufficient to saturate it, and the pressure of the vapour is the maximum pressure for the given temperature.

If, however, saturated vapour be heated, in the absence of its liquid, it expands in accordance with Charles' law, or, if the heating take place at constant volume, its pressure increases in the same way as that of an ordinary gas (see Arts. **52** to **61**). On the other hand, if a saturated vapour be cooled in the absence of its liquid condensation at once takes place, and the vapour remains saturated.

Unsaturated vapours behave as ordinary gases under change of pressure and temperature—that is, so long as they remain unsaturated they obey both Boyle's and Charles' laws. To understand this it should be remembered that, for any given vapour, there is, for each temperature at which it can exist, a definite maximum pressure corresponding to that temperature, and conversely, for any given pressure to which a vapour is subjected, there is a temperature at which this pressure is the maximum. Hence, if an unsaturated vapour, at a given temperature, be subjected to increase of pressure, its volume decreases in approximate accordance with Boyle's law, until a pressure is reached which is the maximum pressure for the given temperature, and therefore at this pressure the vapour becomes saturated, and further increase of pressure produces condensation, as described above. Again, if an unsaturated vapour be cooled, at constant pressure, its volume will decrease in approximate accordance with Charles' law until a temperature is reached for which the given pressure is the maximum,

and therefore at this temperature the vapour becomes saturated, and further cooling produces condensation.

101. Vapour and gas. An unsaturated vapour at temperatures considerably higher than the boiling point of the liquid from which it is derived is usually called a gas. The technical difference between a vapour and a gas turns upon the following point. If an unsaturated vapour be below a certain temperature, known as the *critical temperature* for the substance, it can always be condensed or liquefied by increasing the pressure until the maximum pressure for the temperature is reached. If, however, the vapour be above this critical temperature, it cannot be condensed by increase of pressure however great, and is properly called a *gas*. Hence a gaseous substance at temperatures below its critical temperature is properly called a *vapour*, but above the critical temperature it should be called a *gas*. The critical temperature for carbon dioxide is 30·9° C.; hence above this temperature carbon dioxide is a gas, but below this temperature, if in the vapour state, it is the *vapour* of liquid carbon dioxide.

EXERCISES X.

1. Describe experiments to illustrate the meaning of the term *vapour pressure*. How does the maximum vapour pressure of any substance vary with the temperature?

2. State the laws relating to vapour pressure. How are they experimentally verified?

3. What is meant by a saturated vapour? How does it behave when compressed? and how when allowed to expand in the presence of the corresponding liquid?

4. In what different ways could you convert an unsaturated into a saturated vapour?

5. What becomes of the steam which the boiling water in a kettle discharges into the air?

6. Explain what is meant by air saturated with moisture. What happens if the temperature of such air be (1) raised, (2) lowered?

7. A closed vessel is filled with dry air at a temperature of 15° C. and a pressure of 763·7 mm. of mercury. Without allowing any of the air to escape it is saturated with moisture. The temperature is now raised to 25° C. Find the total pressure.

CHAPTER XI.

CHANGE OF STATE.

Vaporisation and Condensation.

102. Vaporisation and condensation. The process of change from the liquid to the vapour or gaseous state is known as *vaporisation.* The reverse process which takes place when a vapour changes into a liquid is known as *condensation.* The general phenomena of vaporisation and condensation are closely analogous to those of liquefaction and solidification, but the subject is complicated by the fact that a liquid gives off vapour at all temperatures and by the influence of external pressure. The simpler phenomena of the change are illustrated by the following experiments.

Exp. 65. Place some water in a flask fitted with a thermometer and tube as shown in Fig. 63. Arrange the thermometer so that the bulb is well immersed in the water, but not too close to the bottom of the flask. Heat the flask gently and observe the behaviour of the liquid and the readings of the thermometer.

Fig. 63.

As the heating goes on the temperature rises and the vapour which is given off from the surface of the water at all temperatures is observed to condense in the neck and delivery tube of the flask and to form a cloud of condensed vapour at the mouth of the tube. As the temperature rises this escape of vapour at the surface becomes more marked, and small bubbles of air form and escape from numerous points on the walls of the flask. Ultimately, however, the temperature becomes stationary at a fixed front (100° C.), and the liquid becomes agitated by the rapid escape of bubbles of vapour throughout its mass and is said to *boil.* If heating is continued the boiling goes on steadily and the water "boils away";

that is, changes to water vapour or steam; but the temperature remains practically constant throughout the process. If the thermometer be now raised so that its bulb is not in the water but in the steam, it will be found that the temperature still remains constant at the same point. This constant temperature is called the *boiling point* of the water. If the process of boiling is carefully watched it will be seen that the bubbles of vapour escape through the liquid from points on the walls of the flask and that the continuance of the process is facilitated by placing a few pieces of broken glass or scraps of platinum foil in the flask.

Exp. 66. Repeat the above experiment (i) with alcohol and (ii) with ether, taking great care that the vapours do not go near the flame, as they are inflammable and explosive. The general progress of the experiment will be found to be the same for each liquid as for water. Each liquid will, however, be found to have a distinctive boiling point—100° C. for water, about 78° C. for alcohol, and about 35° C. for ether.

From these and former experiments we see that

1. A liquid gives off vapour from its free surface at all temperatures by the process of *evaporation*.

2. A liquid changes by *boiling* or *ebullition* from the liquid state to the vapour state at a temperature which is constant throughout the process of ebullition. This conconstant temperature is the *boiling point* of the liquid under the conditions of the experiment.

3. Under given conditions each liquid has a distinctive *boiling point* characteristic of the liquid.

103. Condensation. During the process of condensation the phenomena of vaporisation are observed in the reverse order. The temperature at which condensation takes place is identical with the boiling point under the same conditions, and just as a liquid in boiling at a given temperature gives rise to vapour at the temperature of the liquid, so a vapour in condensing gives a liquid at the temperature of the vapour. Thus, for example, when water boils at 100° C. the steam produced is also at 100° C., and when this steam condenses the condensed water is, *when first formed*, also at 100° C.

In experimental work it is often necessary to facilitate condensation. This is usually effected by the aid of an

arrangement known as a *condenser*. The simplest form of condenser is the double tube condenser, which consists of an inner tube surrounded by a wider jacket tube. The vapour is passed into the inner tube and a current of cold water is passed through the jacket tube. The cooling effect of the cold water causes the vapour to condense, and the condensed liquid may be run off into a receiver as in Fig. 64, or, the condenser being placed vertical, allowed to run back into the vessel in which the vapour is produced.

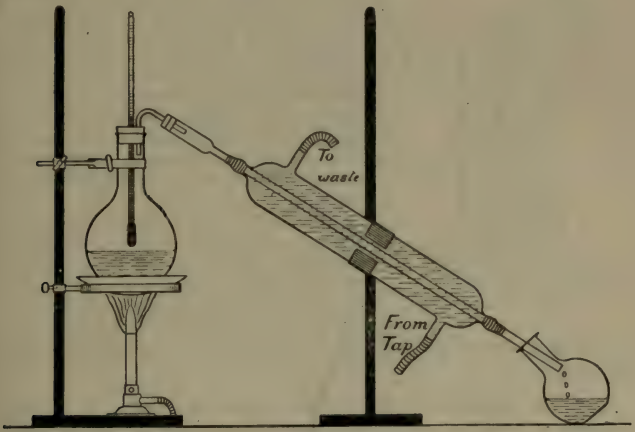


Fig. 64.

The condenser shown in Fig. 64 is commonly known as *Liebig's condenser*. Water or other liquid is boiled in the upper flask and the vapour passes into the inner tube of the condenser. The condenser is kept cold by cold water which enters at the lower end and leaves at the higher end. The condensed liquid trickles down the condenser tube and collects in the lower flask.

Another form of condenser is that in which the vapour is condensed in a spirally coiled tube, known as a *worm*, which is surrounded by cold water.

104. Influence of pressure on the boiling point of a liquid. The quantity of the liquid, the size and material of the flask,* the rate of heating may all be changed without causing any appreciable change in the boiling point, but a change in the pressure to which the surface of the liquid is exposed makes a material change in the boiling point. This fact is simply illustrated by the following well-known experiment due to Franklin.

Exp. 67. Boil water in a strong 20-oz. flask (Fig. 65) until all the air is expelled and its place taken by water vapour. Remove the source of heat, cork the flask tightly, and arrange the flask in an inverted position. Allow the flask to cool in this position until boiling entirely ceases. Then pour some cold water over it so as to condense the water vapour in the space over the water. It will be found that the water again begins to boil vigorously and may be made to do so for some time.

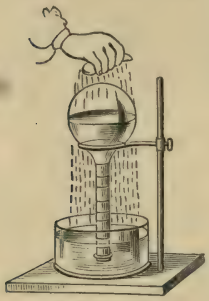


Fig. 65.

The pressure on the surface of the water is evidently decreased by the condensation of the water vapour in the space over it, while it is also evident that the boiling point of the water is lowered, for the water was below 100° C. before the cold water was poured on, and the final temperature is therefore well below this.

Exp. 68. Heat water in a small copper boiler communicating through a condenser with another vessel, in which the pressure may be varied by pumping air into it or out of it. Fit a thermometer into the boiler so that its bulb is well immersed in the steam, and note how the boiling point indicated by it is affected by adjusting the pressure in the air reservoir so that the water boils under pressures *slightly* greater or less than the atmospheric pressure. It will be found that as the pressure increases the boiling point rises, and as the pressure decreases the boiling point falls.

These experiments indicate that the boiling point rises and falls with the external pressure.

* The material of the flask has a slight effect on the temperature of the boiling liquid, but none on the temperature of the vapour arising from the boiling liquid.

It is now necessary to determine the boiling points for a number of measured pressures. There is considerable danger in experimenting with boiling under pressures much greater than the atmospheric pressure, particularly with glass vessels, but for pressures less than the atmospheric pressure investigation may be made safely and conveniently by the method of the following experiment.

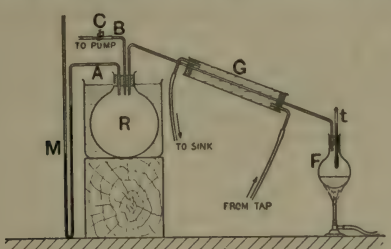


Fig. 66.

Exp. 69. Connect up a flask, F (Fig. 66), through a double tube condenser, C, to a large flask, R, which communicates with a U-tube mercury manometer, M, by a tube, A, and with a water filter pump* by a tube, C, which is provided with a clip. R is kept in a bath of cold water. Its use is to keep the pressure constant by damping out small oscillations in pressure. It also serves to introduce a large volume of air into the apparatus.

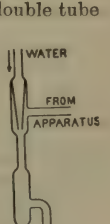


Fig. 67.

The liquid to be experimented on is placed in the flask F, and the pressure under which it is made to boil is adjusted by means of the pump and measured by the manometer M. An accurate thermometer t is fitted into F, so that its bulb is immersed in the vapour of the liquid, gives the boiling point corresponding to each pressure. The vapour produced during boiling condenses in the condenser G, and the condensed liquid runs back into the flask F. The course of an experiment would be

* A water filter pump is a piece of apparatus made as shown in Fig. 67. A stream of water down the centre tube sucks air out of the side tube, and hence, if this side tube communicates to a closed vessel, the air in this vessel is rarefied.

as follows:—Half fill the flask F with the liquid, say water, connect up the apparatus, turn on the water to the pump and condenser and heat the flask gently. When the column of mercury in the manometer tube on the reservoir side rises to a height of, say, 10 cms. or 20 cms. above that on the air side shut off the pump by a screw clip or tap at C. Allow the heating to go on till the water in F boils gently and the reading of the thermometer is stationary. When this is the case, read the thermometer and the height of the manometer column simultaneously and record the readings. Then turn on the pump until the mercury rises another 10 cms. or so and take another pair of readings as before.

This process is continued until a number of readings of boiling points and their corresponding pressures are obtained, the pressure in each case being given by the difference between the height of the barometer and the observed height of the manometer column.

In an experiment of this kind the following readings might have been obtained—

Boiling Point.	Pressure in mms. of mercury.
96° C.	657
92° C.	567
88° C.	486
81° C.	370
74° C.	276
67° C.	204
59° C.	142

A study of these figures brings out a very important result. It will be found on comparison with the table in Art. **97** that *the external pressure for each boiling point is the same as the maximum vapour pressure for that temperature.*

By pumping air into R instead of withdrawing it the temperature of boiling can be observed for pressures greater than atmospheric. Using metal vessels, Regnault determined in this way the maximum vapour pressure of aqueous vapour for temperatures over 100° C.

105. The laws of ebullition may therefore be formally stated as follows :—

1. Ebullition takes place in a liquid at the temperature for which the maximum pressure of the vapour is equal to the external pressure.

2. The temperature at which ebullition takes place, under a constant pressure, remains constant throughout

the process. This temperature is known as the *boiling point* under the given pressure.

The temperature at which ebullition takes place under the normal pressure, which is equal to that due to 760 millimetres of mercury at 0° C. at sea-level in latitude 45°,* is the boiling point usually assigned to a liquid.

3. Under given conditions each liquid has a distinct *boiling point* characteristic of the liquid.

106. These laws may be illustrated by the following experiments :—

Exp. 70. Take a glass tube about a foot long and a quarter of an inch in diameter. Close one end and bend it as shown in Fig. 68, A. Fill the tube with mercury to about 15 millimetres from the open end. Fill the remaining portion with water. Close the tube with the thumb, by inverting the tube pass the water round the bend into the short limb where it rises to the top of the mercury. Now jerk sufficient mercury out of the open limb to bring the level of the mercury in this limb below that of the mercury in the short limb. Now place the tube, as shown at B, in the steam from water boiling in a wide-necked flask. In a short time a portion of the water in the closed limb will be converted into steam, and this steam being saturated will exert the maximum pressure corresponding to the temperature of the surrounding steam. It will then be found that the mercury assumes the same level in both limbs of the tube,

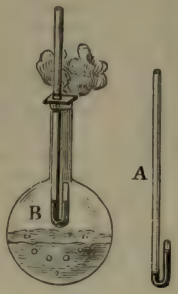


Fig. 68.

* The latitude and height of the place of observation must be given for the pressure due to a liquid column is equal to the product of the height of the column, the density of the liquid and the acceleration due to gravity. The last factor depends on the latitude and height of the place of observation. In London the acceleration due to gravity is greater than that due to gravity in latitude 45°, and it has been calculated that a liquid column at London, 759·6 mm. in height, exerts a pressure equal to that exerted by a column 760 mm. in height in lat. 45°, hence the pressure is normal in London when the barometer (corrected for temperature) reads 759·6 mm.

thus showing that the maximum pressure of water vapour at the boiling point is equal to the pressure under which boiling takes place.*

Exp. 71. Take a siphon barometer tube, introduce sufficient mercury to give the barometric height, and complete the manufacture of the barometer in the usual way. Then add some water to the short limb, draw out the top to a fine hole, and by boiling the water for some time expel all the air from the space over the water and close the end of the short limb by the blowpipe. When the limb is cold a layer of water will rest on the mercury while the space above contains only water vapour. Now surround this limb with a steam jacket tube as shown in Fig. 69. Pass a rapid current of steam from water boiling under the atmospheric pressure through this jacket tube and it will be seen that the steam formed in the wide limb of the barometer tube displaces the mercury into the narrow limb until the difference between the levels in the two limbs is exactly equal to the height of the barometer. This shows that the maximum pressure of the water vapour at the temperature of steam from water boiling under atmospheric pressure is equal to the atmospheric pressure.

Fig. 69.

As the difference in the level of the mercury columns is dependent only on the temperature such an instrument, if well made, proves an effective thermometer.

107. Ebullition or Boiling. The theory of this process can now be more fully considered. Consider a bubble of air in a liquid; its walls are formed by the liquid, and from these walls evaporation goes on until the space which they enclose becomes saturated with vapour. Let V denote the initial volume of air in the bubble, and P the hydrostatic pressure to which it is subjected. The value of P *is determined by the position of the bubble in the liquid*; if it is at a depth *d* below the surface, then P is equal to the atmospheric pressure, *plus* the pressure due to a column of the liquid of height *d*. As evaporation

* Enough water must be placed in the short limb to saturate the space. If more than enough has been inserted a correction must be applied for the water left resting on the mercury in the short limb.

goes on into the bubble, the pressure exerted by the vapour increases, until it becomes equal to F, the maximum vapour pressure corresponding to the temperature of the liquid. But, as the total pressure in the bubble must, so long as the bubble remains in the same position, be equal to P, the pressure of the air in it must, according to Dalton's second law, be equal to (P − F). Hence, if V′ denote the volume of the air after the bubble is saturated with vapour, we have, by Boyle's law—

$$V\,P = V'\,(P - F), \text{ or } V' = \frac{V\,P}{P - F}.$$

From this it is evident that V′ increases as (P − F) diminishes, and becomes infinitely great when P − F = 0; that is, when F = P. This means that when the temperature at any point in any liquid is such, that the maximum vapour pressure for that temperature is equal to the hydrostatic pressure at that point, then if a bubble of air, gas, or vapour form at that point its volume will tend to become infinitely great—that is, it will give rise to an infinite number of bubbles charged with vapour, which rise through the liquid, and escape at its surface. This process of formation of vapour in the interior of a liquid, and its final escape at the surface of the liquid, is called *boiling* or *ebullition*.

If a liquid be placed in an ordinary glass vessel and gradually heated, the temperature rises, and evaporation goes on freely at the surface of the liquid. Before long, however, small bubbles of air are seen to form on the walls of the vessel, and from them minute bubbles are given off, and rise to the surface; as the temperature rises, this process goes on more rapidly, and bubbles are given off freely from the different points on the walls of the vessel, where air bubbles have been seen to form. If the heat is applied from below, the upper layers of the liquid will be colder than the lower ones, and it will be noticed, at a certain stage of the heating, that bubbles given off from points near the bottom of the vessel are condensed in the upper colder layers before reaching the surface. The condensation of these bubbles of vapour is accompanied by

a peculiar noise familiarly known as *singing*. On further heating the temperature of the liquid becomes more uniform, and large bubbles of vapour rise rapidly to the surface and escape. The liquid is thus kept in a state of agitation known as *boiling*.

It thus appears that the commencement of ebullition in a liquid is dependent on the presence of bubbles of air, or other gas, in the liquid or on the walls of the vessel in which it is heated. Experiment shows this to be the case; provided no bubbles of air form, the liquid may be raised to a temperature considerably above its boiling point without entering into ebullition; but if, in any way, a bubble of vapour or any gas is introduced, boiling at once takes place with explosive violence.

On the other hand, the presence of a very minute quantity of air or other gas in a liquid is sufficient to produce continuous boiling as long as any liquid is left. This can be shown by dropping a small glass bulb containing air and drawn out to a very fine hole into a liquid heated to its boiling point; the air in the bulb will give rise to thousands of vapour bubbles without being appreciably diminished in volume.

The same purpose is also served by the fragments of charcoal, porous pot and broken glass used to steady the boiling of liquids in many chemical and physical operations. See Art. **54** (footnote).

108. Papin's digester. If heated in a closed vessel water may be raised to very high temperatures. This fact is often of great utility in the industrial arts. An instance of its application is found in **Papin's digester**, which is used for the purpose of subjecting substances to the action of water at a temperature considerably higher than 100° C. It consists of a strong bronze vessel, covered with a lid secured by a screw and fitted with a safety valve, to prevent the internal pressure becoming greater than the vessel can sustain. Again in the boiler of a steam engine the temperature rises considerably if the steam is not drawn off. For example, if the gauge registers 55 pds. per sq. in. the temperature within is about 150° C.

109. The hypsometer. This instrument (Fig. 70) is merely a small vessel conveniently constructed for observing the temperature of ebullition at any place. It consists of a small boiler heated by a spirit lamp, and is provided with a delicate thermometer which is surrounded by steam in the upper part of the vessel. The steam escapes at the top by a small side tube which communicates with the outer air, so that ebullition takes place under the atmospheric pressure. The hypsometer is used for the determination of heights, and is very convenient for this purpose because of its portability. The principle of its use for such determinations is based on the fact, that the temperature of ebullition depends on the external pressure, and that when the former is known the latter is determined, and is equal to the maximum vapour pressure at that temperature; hence, if we find the temperature at which water boils at the sea-level, and at a place the height of which we wish to determine, we can, by reference to a table of maximum pressures of aqueous vapour, find the corresponding atmospheric pressures, and when these are known the required height can be calculated. It is found that the boiling point of water falls 1° C. for every 1,080 feet increase in elevation. According to Soret's formula, the height (h) of any place above the sea-level is approximately given by $h = 295\ (100 - t)$, where t is the temperature at which water boils at that place, and h the required height in *metres*. The fact that water boils below 100° C. at high elevations is a source of inconvenience when cooking has to be done in such places. In such cases a modified form of Papin's digester is used, a saucepan with a screw cover provided with a thermometer being an easy solution of the difficulty.

Fig. 70.

Example.—The boiling point of water on the top of Snowdon = 96·3° C. Find the height of the mountain. Applying the above formula we get the height

$= 295 \times 3{\cdot}7$ metres $= 1091$ metres $= 3580$ feet.

110. Determination of the boiling point of a liquid.

Exp. 72. Determine the boiling points of various liquids such as ether, alcohol, turpentine, acetic acid. Take a clean, rather small flask, F (Fig. 71), preferably provided with a side tube, *t*, and fit a good thermometer through a cork which just fits the neck of the flask.* Place some glass fragments in the flask to facilitate boiling, and then nearly half fill the flask with the liquid. Mount the flask over a Bunsen burner, the flame of which is protected from draughts by a chimney, and heat the liquid to boiling. When the liquid is boiling adjust the thermometer so that the top of the mercury column is just above the cork, taking care, however, that the bulb is not immersed in the liquid. As soon as the temperature of the thermometer becomes steady read it; it is the boiling point at the pressure inside of the flask. If the boiling has not been too rapidly carried on the inside pressure is equal to the atmospheric pressure which may be read, and the boiling point corrected to a pressure of 760 mm. by means of tables.

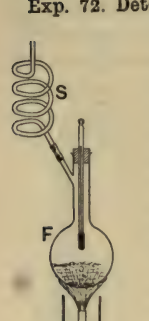


Fig. 71.

If the vapour must not be allowed to escape join the end tube to a length of open spiral tubing S, in which the vapour may condense and flow back again into the flask, or join the exit tube to a condenser (Art. **103**).

If the liquid is obtainable only in very small quantity, the flask may be replaced by a test tube as in **Exp. 6.**

TABLE OF BOILING POINTS AT 760 MM. PRESSURE.

Liquid air	−191° C.	Benzene	80·4 C.
Liquid carbonic acid	−78·2	Distilled water	100·0
Liquid ammonia	−38·5	Spirits of turpentine	130·3
Liquid sulphur dioxide	−10·0	Oil of turpentine	159·1
Ether	35·0	Sulphuric acid	325·0
Chloroform	60·2	Mercury	356·7
Alcohol (ethyl)	78·3	Sulphur	443·6

* Many text-books insist on a double walled tube being used, the thermometer being inside and vapour in contact with both sides of the inner wall. There is, however, no necessity for this.

111. Evaporation. It has been shown that, at all temperatures, vaporisation takes place in a closed space until that space is saturated with vapour. If, however, the process goes on in the open air it becomes continuous, because it is practically impossible to saturate the atmosphere with vapour. This process of vaporisation in an unlimited atmosphere has been called *evaporation.*

Evaporation takes place exclusively at the free surface of the liquid. The vapour formed at the surface penetrates into the adjacent layer of air, and from thence it passes into the next layer, and so on until the vapour is diffused throughout the atmosphere at a uniform rate such that any given layer passes on as much vapour as it receives. This process is somewhat modified if the air is in motion, for then the layer in contact with the liquid is continually being renewed, and evaporation takes place more rapidly, because each layer as it comes in contact with the liquid takes up only a small quantity of vapour, which is at once carried away and thus diffusion is assisted; moreover evaporation always takes place into a comparatively "dry" atmosphere, for the successive layers pass on before becoming nearly saturated.

Evaporation is also rapidly promoted if a rapid succession of air bubbles be blown through the liquid, for each air bubble exposes a large surface to the liquid and is then immediately swept away.

The following experiments illustrate some of the more important phenomena of evaporation:—

Exp. 73. Put a small quantity of ether into a watch glass and set it on the table. In a few minutes the ether will have disappeared by evaporation.

Exp. 74. Dip a piece of filter paper or a handkerchief in water and hang it out to dry. In a short time, which depends greatly on the weather, the water will have "dried" out of the fabric by evaporation.

Exp. 75. Put equal quantities of water, alcohol, and ether in similar crystallising dishes and allow them to evaporate under the same conditions. The ether evaporates more rapidly than the alcohol, and the alcohol more rapidly than the water.

Exp. 76. Put equal quantities of water (or alcohol or ether) in crystallising dishes of different diameters and allow them to evaporate under the same conditions. Note the times taken for the dishes to completely dry up. The amount of liquid evaporating per second will be found to be proportional to the diameters, and not as one would suppose on first thoughts, viz. proportional to the surfaces.

Exp. 77. Put equal quantities of ether in two equal crystallising dishes. Direct a gentle current of air from a foot bellows over the surface of the ether in one of the dishes. The ether in this dish evaporates much more quickly than in the other.

Exp. 78. Put equal quantities of alcohol (methylated spirits will do) into crystallising dishes of the same size, and expose one to evaporation in a warm living-room and the other in a cool cellar, but under otherwise similar conditions. Evaporation is much more rapid from the former than from the latter.

From the above experiments we conclude that different liquids evaporate at different rates and that evaporation can be greatly quickened by (1) renewing the air in contact with the surface, (2) warming the liquid and air, (3) increasing the surface of the liquid exposed to the air.

The evaporation of water is a familiar and important process. All the aqueous phenomena of meteorology have their origin in the evaporation which goes on from the water surfaces of the earth into the atmosphere. Such processes are so apparent that it is hardly necessary to call them to mind. Some of them will, however, be dealt with in detail in Chapter XIII.

112. Conditions favourable to evaporation. From what has been said above, it may readily be deduced that the rate at which evaporation takes place is favoured by—

1. Dryness of the air. The less the quantity of vapour present in the air the more readily does evaporation go on into it.

2. Low atmospheric pressure. Low pressure facilitates the escape of vapour from the surface of the liquid.

3. High temperature both of the liquid and the air. The higher the temperature of the liquid the greater the pressure exerted by the escaping vapour, and the higher the temperature of the air the greater the quantity of vapour it will contain before saturation is reached.

4. Large extent of free surface of the liquid.

5. Low boiling point of the liquid. The lower the boiling point the greater the pressure of the vapour at ordinary temperatures.

6. Renewal of the air in contact with the surface of the liquid.

113. Cold caused by evaporation. During evaporation the latent heat necessary to effect the required change of state is absorbed from the liquid itself and its surroundings. This absorption of heat cools the liquid, but, as evaporation takes place at all temperatures, the process goes on, and a continuous fall of temperature is thus produced. Under favourable conditions this fall of temperature may go on until the freezing point of the liquid is reached and the liquid freezes. The following experiments illustrate this cooling effect :—

Exp. 79. Put a little ether in a thin watch-glass (or in a small thin beaker), and set it up on a wood block with a small drop of water between it and the block. Blow a rapid stream of air through the ether. Rapid evaporation is promoted, the ether in turning from liquid to vapour absorbs its latent heat from the glass, wood, and film of water between, and after a short time so much heat will have been abstracted that the water will freeze and the watch-glass and block form one piece.

Exp. 80. Place a shallow dish containing a little water, and another dish, containing strong sulphuric acid, under the receiver of an air pump. Exhaust. The water rapidly evaporates, and the vapour formed is absorbed by the sulphuric acid, thus ensuring continuous and rapid evaporation of the water. This process cools the water, and ultimately a thin layer of ice forms on its surface. This experiment is usually known as *Leslie's experiment*.

114. Pressure of vapour in a system of communicating vessels at different temperatures. Let A and B (Fig. 72) be two bulbs connected by a tube C. Suppose further that A and B both contain some of the liquid and that the remaining space contains only the vapour of that liquid. Let A and B be placed in baths of temperatures t_1 and t_2, t_1 being greater than t_2 and

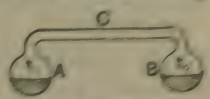


Fig. 72.

suppose p_1 and p_2 to be the maximum vapour pressures at t_1 and t_2, p_1 of course being greater than p_2. Suppose a closely fitting piston to be placed in C. Then since p_1 is greater than p_2 the piston will be forced towards B. As it moves towards B it tends to increase the pressure in B, but since the pressure cannot rise above p_2 some of the vapour in B will be condensed. Similarly its movement away from A causes some of the liquid in A to evaporate. This action proceeds whether the piston is there or not, and so we see that the effect of A and B being at different temperatures is that there will be a continual transference of vapour from A to B until all the liquid in A has evaporated and the pressure throughout the vessel is p_2, the maximum vapour pressure corresponding to the lower temperature t_2.

This explains the action of Wollaston's cryophorus (Art. **115**) and the possibility of distillation (Art. **116**).

115. Wollaston's cryophorus. A form of apparatus known as the *cryophorus* was devised by Wollaston to illustrate the cooling effect of evaporation. It consists of a bent tube (Fig. 73) with a bulb at each end. It contains only a small quantity of water and water vapour. All the water being passed into the bulb B, which should now be not more than half full, the bulb A is surrounded, as shown in the figure, by melting ice. The cold surface of the ice condenses the vapour in A as rapidly as it is formed in B, and in a short time the absorption of heat is sufficient to cool the water in B down to the freezing point and finally convert it into ice.

Fig. 73.

116. Condensation and distillation. Distillation is a combined process of vaporisation and condensation; it is much used for obtaining pure or distilled water from ordinary tap water, or from salt water. It is also of frequent use for manufacturing and chemical purposes. The liquid to be distilled is heated in a suitable vessel and

the vapour produced passes through a long spirally coiled tube, surrounded by cold water, or is, in some other way, cooled to a temperature sufficiently low to cause the vapour to condense (Fig. 64). The liquid thus produced is collected in a vessel into which the worm opens; it is free from all solid matter contained, either in suspension or solution, in the original liquid.

117. Sublimation. Some substances do not melt, but "sublime," *i.e.* pass direct from the solid state to the vapour state, and *vice versa.*

Exp. 81. Take a small piece of iodine, place it in a dry test tube, and heat gently in a flame. It is easily seen that the iodine does not melt, but passes off into a violet vapour, which soon fills the tube: some of this vapour will condense again on the cooler portions of the tube, forming a thin layer of the solid. The same process takes place more slowly at ordinary temperatures, as may be at once verified by examining a bottle in which iodine has been kept for some time.

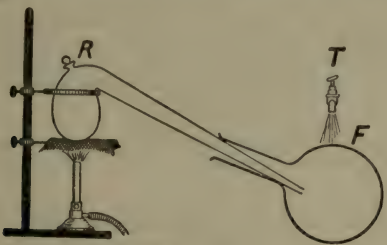


Fig. 74.

Exp. 82. Place some bar sulphur in a retort, *R*, and lead the stem of the retort into a large flask, *F*, over which a stream of water flows to keep the interior cool. Heat the sulphur, and it will be found that sulphur vapour passes over into the flask and there condenses, forming a yellow cloud of solid particles which collect on the sides of the flask, forming a layer of "flowers of sulphur."

It is known that ice and snow evaporate at temperatures below the freezing point. Snow often disappears completely by evaporation on very cold days, when no trace of thawing can be detected. This is the only kind of evaporation which can take place in the Arctic regions.

EXERCISES XI.

1. What is meant by the "boiling point" of a liquid? How is it affected by a change of pressure?

2. A flask containing water is heated. When the water boils, the flask is carefully closed with a cork and removed from the flame. Explain why, when the flask is dipped into cold water, the water inside begins to boil again.

3. Explain why, in order to cook food by boiling at the top of a high mountain, you must employ a different method from that used at the sea level.

4. A beaker containing water is heated by a Bunsen flame. A Centigrade thermometer, placed in the water, rises to 100°, but no higher, and the water begins to boil. What is the reason that the thermometer does not rise higher than 100°, and what becomes of the heat which is thus apparently lost?

5. What atmospheric conditions are favourable to the rapid drying of muddy roads?

6. I once went into a room the doors and windows of which had been kept shut for some time, and the temperature of which was 80° F. I took some water (also at 80°) and sprinkled it over the floor, and the temperature at once fell several degrees. How do you explain this?

7. If you drop a liquid which readily evaporates on your hand, you feel cold. What is the reason?

8. If you dip your hand in lukewarm water and then expose it to the air, the hand feels cold. If you make the same experiment with ether, the hand feels much colder on exposure. Explain these facts.

9. Explain what is meant by air saturated with moisture. What happens if the temperature of such air be (i) raised, (ii) lowered?

10. A flask with its neck drawn out, half filled with water, has the air above the water removed by an air-pump; the drawn-out neck is then sealed by a spirit-lamp, and the flask is plunged into water of not more than a blood-heat. On lifting it out it boils violently. Explain this.

11. When the attempt is made to boil eggs at the top of a high mountain it is found that, if the eggs are immersed in boiling water for the same time as is required to cook them at the sea level, they are not equally cooked. Explain carefully the reason for this failure.

12. Name the three states in which matter can exist, and the cause to which changes from one state to another are chiefly due Give two experiments illustrating these changes of state.

13. Explain the action of Wollaston's cryophorus.

14. What is a hypsometer and for what purpose is it used?

15. If the maximum pressure of water vapour is 758 mm. at 99·93° C. and 759 mm. at 99·96° C., what increase in size takes place in a bubble of air just under the surface of some water as the temperature rises from 99·93 to 99·96, supposing the pressure of the atmosphere to be 760 mm.? What further increase would take place as the temperature rose from 99·96° C. to 100° C.

16. A quantity of hydrogen is collected, over water, in a eudiometer tube. The height of the column of water, left in the tube, is 40·8 mm., and its temperature is 15° C.; find the pressure of the hydrogen in the upper part of the tube. (Take the height of the barometer at 758 mm.) The space occupied by the hydrogen is saturated with aqueous vapour.

CHAPTER XII.

THE LATENT HEAT OF VAPORISATION.

118. When water is boiled it has been shown that the temperature of the steam produced is the same as that of the boiling water. Where then has the heat given to the water by the burning gas gone? On comparing this process with that observed when ice melts it is evident that the heat has been used up in effecting the change of state from liquid to vapour, *i.e.* the heat has been rendered "latent," to use the same word as we have used before.

This heat is given out in the process of condensation of steam to water, as is shown in the following experiment.

Exp. 83. Place about 250 c.c. of cold water in a beaker, insert a thermometer, and weigh the whole. Now boil some water in a flask fitted with a delivery tube, and pass the steam given off into the water in the beaker.

As the bubbles of steam enter the cold water they condense with a crackling sound and the temperature of the water rises rapidly. By weighing the beaker we ascertain the weight of the steam condensed.

This does not by itself prove that steam gives out heat in *condensing*. When the steam enters the cold water we may consider that the following changes take place:—

(1) It condenses into boiling water without change of temperature.

(2) This boiling water mixes with the cold water in the beaker.

Hence, if steam gives out heat in condensing simply, the rise of temperature produced by passing the steam into the cold water should be greater than that produced by mixing the same quantity of boiling water with the cold water.

If now we take the same quantity of water in the same beaker with the same temperature indicator and add to it a quantity of boiling water equal to that obtained by the condensation of the steam, it will be found that this gives a much smaller rise of temperature. This shows that the steam, in condensing from steam to boiling water without fall of temperature, gives out a considerable quantity of heat.

119. Latent heat of vaporisation. It is found that a given liquid absorbs, per unit mass, during vaporisation and evolves, per unit mass, during condensation a definite quantity of heat which is constant for the same substance under the same conditions. The quantity of heat required to convert *unit mass of a substance* from the liquid state at the boiling point to the vapour state at the same temperature is called the **latent heat of vaporisation** of that substance. Thus the latent heat of vaporisation of water at 100° C. is the quantity of heat required to convert unit mass of water at 100° C. into steam at 100° C. It is also the quantity of heat given out by unit mass of steam at 100° C. in condensing to unit mass of water at 100° C. If the *gramme* be taken as the unit of mass, then the latent heat should be expressed in *gramme* degrees; that is, the same unit of mass should be involved in both measurements.

It follows that if L denote the latent heat of vaporisation of any substance, then the quantity of heat absorbed during vaporisation or evolved during condensation by m grammes of the substance is mL units.

It is to be noted that the latent heat of vaporisation of any substance depends upon the temperature at which vaporisation takes place. If the conditions of vaporisation change the boiling point it is to be expected that the latent heat of vaporisation will also change.

Exp. 84. Find roughly the latent heat of vaporisation of steam.

Method I. (No thermometer is required.) Place in a can a mixture of ice (one part by weight) and cold water (about four parts by weight). Arrange a Bunsen burner under a tripod, keep it burning steadily throughout the experiment. When all the ice in the mixture has melted note the time and at once place the can over the Bunsen flame. Note also the time when (ii) boiling begins, (iii) the water has just boiled away.

Method II. (A thermometer is required.) Put a can, half filled with water, over a Bunsen burning steadily. Note the time when (i) the temperature is, say, 25°, (ii) boiling begins, (iii) the water has just boiled away.

Calculation. Assume that the heat absorbed from the flame by the water is proportional to the time during which it has been heated. Find (i) the time, T, taken to boil away the whole of the

water, (ii) the time, t, taken to warm the water from ice-point to boiling. In Method I. the initial temperature is the ice-point, in Method II. it is, say, 25°. In the latter case calculate the whole time it would have taken to heat the water from 0° to 100° if the initial temperature had been 0°. Finally

$$\frac{\text{“Latent heat” absorbed in vaporisation}}{\text{“Sensible heat” absorbed from ice- to steam-point}} = \frac{T}{t}.$$

Example. Time taken to heat a quantity of water from 25° to 100° = 3 mins. 30 secs. Time taken to boil away the water = 23 mins. 40 secs. Then time taken to heat the water from 0° to 100° = $\frac{4}{3} \times 210$ = 280 secs. ∴ (latent heat) / (sensible heat) = 1420/280 = 5·1. *i.e.* the latent heat of vaporisation of water = 510 calories per gramme.

120. Determination of the latent heat of steam. The latent heat of vaporisation of water at 100° C. is usually called the latent heat of steam. To determine this constant a calorimeter is prepared in the usual way by placing in it a known quantity of water at a known temperature. A current of *dry* steam from a boiler is then passed into the water in the calorimeter. As the steam condenses in the water latent heat is evolved and the temperature rises. When a sufficient rise of temperature is produced the current of steam is stopped and the temperature of the water in the calorimeter noted. The calorimeter is then weighed and the increase of weight gives the mass of the steam condensed. As the experiment lasts only a few seconds, the rise of temperature being very rapid, the cooling correction is not of great importance, but it is well to arrange that the mean temperature of the calorimeter is the same as the temperature of its surroundings. The great difficulty and a great source of error in this experiment is the difficulty of ensuring that the steam which enters the calorimeter is *dry*. It is obvious that if the steam carries with it a fine spray of condensed water the heat evolved in the calorimeter will be much less than if only dry steam entered the calorimeter. Various devices for “trapping” or jacketing the current of steam have been adopted for the purpose of *drying* it; that is, removing the fine drops of condensed water in it.

Exp. 85. Find the latent heat of vaporisation of water. Fig. 75 shows a useful form of apparatus. Water is boiled in the vessel *K*, the steam is led through a bent glass tube (which should be covered with cotton wool) into the calorimeter *E*, hung in the screening can *F*. The outside of *E* and the inside and outside of *F* should be bright. Sometimes a trap, like Fig. 76, is placed at the delivering end of the steam tube. It catches the steam that condenses in the

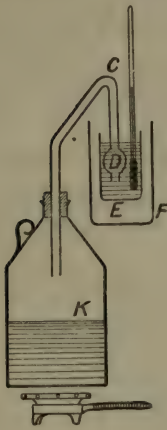


Fig. 75.

Fig. 76.

tube. It should be very well wrapped in wool and provided with a jet and pinch-cock, *R*, to draw off the condensed water just before passing steam into the calorimeter.

Boil *K* briskly. Weigh the calorimeter empty and again when two-thirds full of water. Use a sensitive thermometer: carefully stir the water with it and note the temperature.

It is advisable to have the initial temperature of the water as much below the air temperature as the final temperature is above it. This obviates any correction for gain of heat from outside sources or loss of heat to them. A preliminary experiment will supply the best temperature at which to work.

Place the steam-jet underneath the surface of the water and stir gently with the thermometer. The temperature rises rapidly. Withdraw the calorimeter from the jet when the temperature has reached, say, 35° C. Stir well. Quickly note the temperature. Remove the thermometer from the water, letting its bulb trail

along the inside (above the water) of the calorimeter, so that any drops of water adhering to the thermometer may be removed from it. Weigh. The increase over the last weighing gives the weight of steam condensed. Assume that the temperature of steam is 100° C., and work out the results as indicated above.

Example. *Determine the latent heat of vaporisation of water from the following observations :—*

Mass of copper calorimeter = 52·94, mass of calorimeter plus water = 192·14 gms., mass of calorimeter plus water plus condensed steam = 197·12 gms.

Temperature of steam 100° C., of cold water at the beginning of the experiment 14·4° C., of water finally 35·4° C. Air temperature throughout equals 21° C.

The heat given out

(1) by the 4·98 grammes of steam at 100° C. condensing to water at 100° C. = 4·98 L calories,

(2) by the 4·98 grammes of water cooling from 100° C. to 35·4° C.
$= 4{\cdot}98\ (100 - 35{\cdot}4) = 4{\cdot}98 \times 64{\cdot}6$ calories.

The heat absorbed

(1) by the 139·2 grammes of water in rising from 14·4° C. to 35·4° C.
$= 139{\cdot}2 \times 21{\cdot}0$ calories,

(2) by the 52·94 grammes of copper in rising from 14·4° C. to 35·4° C.
$= 52{\cdot}94 \times {\cdot}095 \times 21{\cdot}0 = 5{\cdot}03 \times 21{\cdot}0$.

Equating the heat given out to the heat absorbed we get

$$4{\cdot}98\,L + 4{\cdot}98 \times 64{\cdot}6 = (139{\cdot}2 + 5{\cdot}03) \times 21{\cdot}0.$$

$$\therefore\ L = \frac{144{\cdot}2 \times 21{\cdot}0}{4{\cdot}98} - 64{\cdot}6 = 545 \text{ calories}.$$

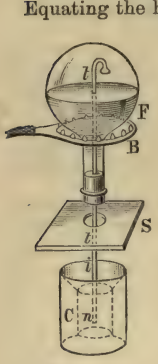


Fig. 77.

Accurate results are hard to get with this elementary apparatus. A better form of apparatus is that of Fig. 77. The flask *F* (or tin as above) is inverted and heated by a ring burner *B*. The delivery tube, *tn*, passes through hot water for a great part of its length and then straight into the calorimeter *C*, which is lifted up during this operation. Direct radiation from boiler to calorimeter is prevented by a screen *S*. With this apparatus a result L = 540 can be obtained. If other liquids are being used, either as vapour or in the calorimeter, a condensing worm is used to keep the condensed

vapour by itself. The increase in weight of the worm gives the weight of the vapour condensed: the calculation is similar to that in the case of water.

The latent heat of vaporisation of water is therefore 540 calories, *i.e.* one gramme of steam at 100° C. in condensing to water at 100° C. gives out 540 calories, or one gramme of water at 100° absorbs 540 calories in becoming steam at 100° C.

TABLE OF LATENT HEATS OF VAPORISATION.*

At boiling point under normal pressure.

Water	540
Sulphur	362
Alcohol	202
Ether	90
Oil of turpentine	74
Liquid air	55

121. Steam calorimeter. The latent heat of steam may be applied, just as the latent heat of water is applied in the ice calorimeters, to ascertain the specific heat of a substance. The invention and perfection of the steam calorimeter is due to Professor Joly. The general principle applied in a steam calorimeter is as follows:—If a piece of any substance at a known temperature be suspended in an empty vessel and steam at 100° C. be passed into the vessel, a mass of steam just sufficient to raise the temperature to 100° C. will condense on the substance, and this mass will remain constant so long as the substance remains in the atmosphere of steam. Loss by the dropping of the condensed water from the substance must of course be guarded against. Hence, if the mass of water which condenses on the substance can be determined, the specific heat of the substance can be calculated.

Example. If a lump of copper of mass 100 grammes and temperature 20° C. is suspended in the steam chamber and 1·45 grammes of steam are found to condense on it, find the specific heat of copper.

* The latent heat decreases as the temperature of boiling rises.

The heat given out by the steam is 1·45 × 540 units or 783 units. This heat is absorbed by the copper in heating from 30° C. to 100° C. Therefore if s denote the specific heat of copper, we have $100 \times 80 \times s = 783$ or $s = ·098$.

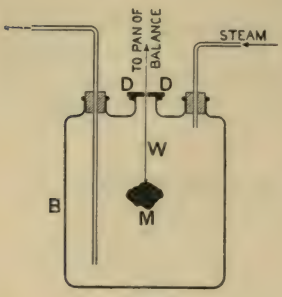


Fig. 78.

The steam calorimeter as designed by Prof. Joly is somewhat complicated. The simple form described below gives fairly satisfactory results. Take a Wolff's bottle, B (Fig. 78), provided with three holes; fit the side holes with steam delivery and exit tubes. The central hole must be ground perfectly level so that a thin disc of aluminium, D D, rests evenly and moves easily on it. The body, M, to be suspended in the steam hangs by the wire, W (preferably of platinum), which passes through a small hole in the centre of the aliminium disc. In the course of the experiment the body is weighed first in air and then in steam, and the difference of the weights * gives the weight of water condensed on the body. It will be seen that, if the aluminium disc moves freely, it will adjust its position so that the wire W always passes freely through the hole in the disc. Care should be taken that water is not deposited on the wire and in the hole.

122 Change of volume accompanying change of state from liquid to vapour. Let any liquid boil and notice the large quantity of vapour produced. It is evident that when a liquid changes to a vapour there is a very great increase in volume. In the case of water, for example, the volume of the steam at 100° C. and atmospheric pressure is nearly seventeen hundred times the volume of the water from which it is produced; that is, a cubic inch of water produces about a cubic foot of steam.

This large expansion accounts in some degree for the large latent heat of vaporisation, for during the change of state the rising vapour does work in pushing away the atmosphere, and, as shown later in Ch. XVII., heat and work are mutually convertible.

---

* For accurate result the difference in weights should be corrected in the first case for the buoyancy of air and in the second for the buoyancy of steam.

The following curve shows the changes in volume which occur when 1 gramme of ice at − 20° C. is heated to steam at 110° C.

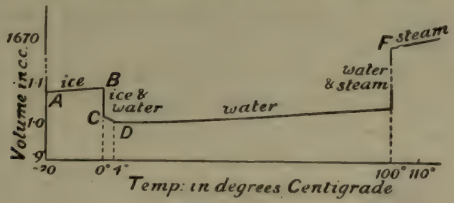


Fig. 79.

The curve is plotted from the following data:—

Temperature.		Volume.
− 20°	Ice	1·0900
0	Ice	1·0908
0	Water	1·0001
4	Water	1·0000
50	Water	1·0120
100	Water	1·0432
100	Steam	1672
110	Steam	1716

The steam line is of course directed towards a point on the temperature axis situated at 273° C. below zero.

EXERCISES XII.

1. Explain exactly what you mean by the latent heat of water and the latent heat of steam.

2. What difference would it make to the living creatures in ponds and rivers if the specific heat and the latent heat of water were greatly diminished?

3. A beaker containing water is heated by a Bunsen flame. The temperature rises to 100° C., but no higher. What becomes of the heat which is thus apparently lost?

4. How many lbs. of steam at 100° C. will just melt 35 lbs. of ice at 0° C. ?

5. How many lbs. of steam at 100° C. will raise 12 lbs of water from 15° to 90° C. ?

6. If $\frac{1}{2}$ lb. of steam at 100° C. is blown into 30 lbs. of water at 12° C., find the resulting temperature of the water.

7. Express the latent heats of steam and water in terms of the degree Fahrenheit.

8. 10 grammes of steam at 100° C. are blown into 100 grammes of ice-cold water at 0° C. Find the rise of temperature produced. The water equivalent of the calorimeter is 15 grammes.

9. 50 grammes of steam at 100° C. are passed into a mixture of 100 grammes of ice and 200 grammes of water at 0° C. Find the rise of temperature. Water equivalent of calorimeter is 15 grammes.

10. Into 200 grammes of water at 20° C. are poured 100 grammes of pounded ice at 0° C. Steam is then passed into the mixture until all the ice has melted and the temperature of the water is again 20° C. On weighing, the increase is found to be 16·2 grammes. Find the latent heat of vaporisation of water.

11. Find the latent heat of steam at 100° C. from the following data :—

Weight of calorimeter	105 gms.
„ „ „ and water . . .	346 gms.
Initial temperature of calorimeter and water .	4° C.
Final „ „ „ „ „ .	24° C.
Weight of calorimeter, etc., at the end of the experiment	354·16 gms.
Water equivalent of calorimetric apparatus .	9 gms.
Height of barometer	752 mm.

12. Find the specific heat of quartz from the following data observed in an experiment with the steam calorimeter: 16·73 grammes of quartz were hung in a chamber at 15·25° C. Steam at 100·50° C. was then passed into the chamber and the water condensed on the quartz weighed ·497 gramme.

13. Explain why it is that if steam at 100° C. be bubbled through a solution of calcium chloride the temperature can be raised to 108° C.

CHAPTER XIII.

HYGROMETRY.

Formation of Clouds, Fog, and Dew.

123. Water vapour in the air. Evaporation is constantly going on from the surface of the sea, rivers, and lakes. There must therefore always be more or less water vapour in the air. This water vapour when quite free from small particles of condensed water is invisible, but we may convince ourselves of its presence by one or two simple experiments.

Exp. 86. Fit a flask with a delivery tube, as shown in Fig. 80, and boil some water in it. A steady current of water vapour obviously issues from the delivery tube and mixes with the air in the room. Water vapour must therefore be diffused throughout the air in the room, although, with the exception of the small cloud of condensed vapour at the mouth of the tube, there is no visible sign of its presence.

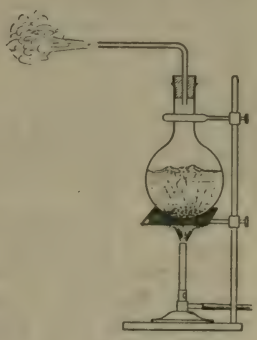


Fig. 80.

Note in this experiment (*a*) the *dry* water vapour in the flask and in the tube is invisible, (*b*) the jet of steam issuing from the tube is invisible until it is condensed by contact with the colder air to form the small cloud which appears *at a short distance* from the mouth of the tube, (*c*) the cloud consists of minute drops of water, (*d*) the water drops of the cloud quickly evaporate and the vapour diffuses throughout the air in the room.

Exp. 87. Fill a glass vessel full of cold water and after thoroughly drying the outer surface of the vessel bring it into a "living room" or into a room where the preceding experiment is going on. Beads of moisture, resulting from the condensation of the vapour in the air, will quickly form on the surface of the cold glass.

124. Relative humidity. We speak of the air as being *dry* or *moist* according as we think it contains little or much moisture, but the condition of the air, in relation to dryness or moisture, involves two elements: (1) the quantity of vapour *actually present* in the air; (2) the quantity of vapour necessary to *saturate* the air under the same conditions, and it is on the *ratio* of these two elements that our sensations of dryness and moisture chiefly depend and not on the first alone. Thus, the air in a warm room may really contain more aqueous vapour than the outside air and yet be *drier* because the amount required to saturate it is so much greater; for we have seen that the mass of vapour required to saturate a given space increases with the temperature, and is independent of the presence of air or other gas in that space. **The hygrometric state of the air, considered as the ratio of the mass of aqueous vapour actually present in a given volume to the mass of vapour required to saturate the same volume, at the same temperature,** has been called the **humidity** or **relative humidity** of the air, and is usually expressed either as a ratio or as a percentage.

Example. If a cubic metre of air at a particular temperature contain 10 grammes of aqueous vapour, and if 30 grammes are required to saturate it at the same temperature, the relative humidity is $\frac{1}{3}$, or 33 , if expressed as a percentage.

125. Determination of relative humidity by the absorption method. If we can find by actual experiment the mass of vapour really present in a given volume of air, and also calculate the mass of vapour required to saturate the same volume at the same temperature we can at once find the humidity of the air. For this purpose a quantity of the air is drawn through a series of drying tubes containing fragments of pumice-stone soaked in concentrated sulphuric acid. To effect this the drying tubes are connected together

by short communicating tubes, and one end of the series is connected to the upper tube of an aspirator,* while the other opens into the outer air. The aspirator is filled with water, and, when the communication with the drying tubes is effected, the upper and lower stopcocks are opened. The water then flows out by the lower tube and air is sucked in through the drying tubes, and is collected in the upper part of the aspirator. The air, in passing through the drying tubes, is deprived of its moisture, and by determining the increase in weight of the first two or three tubes, the weight of moisture deposited is known. The volume of the air drawn through the apparatus is determined by the weight of the water which has escaped from the aspirator.† This volume being determined, it only remains to calculate, from Regnault's tables, the mass of vapour necessary to saturate it at its initial temperature, and then deduce the relative humidity from the ratio—

$$\frac{\text{Quantity of vapour actually present in the given volume.}}{\text{Quantity of vapour that would saturate this volume at the existent temperature.}}$$

126. Dew-point. Under ordinary circumstances the quantity of water vapour present in the air is not enough to produce saturation, and the pressure due to the vapour present is therefore less than the maximum vapour pressure for the temperature of the air. If, however, the air be gradually cooled a temperature is soon reached at which the amount of vapour actually present is sufficient to produce saturation. This temperature is called the *dew-point*. At the dew-point the air is saturated with the vapour present, and the pressure due to the vapour is therefore the maximum vapour pressure at that temperature. When the air is cooled below the dew-point

* The aspirator, Fig. 81, A, is a large vessel of from 5 to 10 litres capacity, having a tube fitted with a stopcock at its upper and lower ends.

† If great accuracy is required this volume must be corrected for the temperature of the water in the aspirator and the aqueous vapour present. (See Art. **134**.)

condensation *begins* at the dew-point, and as the temperature falls the excess of vapour over the amount necessary to give saturation at any temperature condenses out. If the air is cooled by contact with the surface of a cold body the layer of air in contact with the surface quickly reaches the dew-point, and a film of condensed moisture or dew is deposited on the cooling surface. If, however, the air is cooled, as a whole or in part, without being in contact with a cooling surface on which the condensed moisture may be deposited, a slight cloud or fog of minute drops of condensed water may form in the cooled air. (Arts. **135**, **136**.)

127. Determination of relative humidity from a knowledge of the dew-point. The absorption method described above leaves nothing to be desired so far as accuracy is concerned, but it is a somewhat tedious and difficult process; and, as it is very often necessary, for meteorological and other purposes, to know the hygrometric state of the air at the actual instant considered, and not the mean hygrometric state during, say, an hour, simpler methods, based upon the determination of the *dew-point*, have been adopted.

Since the mass of water vapour present in a given volume of air at saturation is very nearly proportional to the saturation pressure, we may also adopt the following as a definition of relative humidity:—

The *relative humidity* of the air at a given instant is equal to the ratio of the maximum vapour pressure at the temperature of the dew-point to the maximum vapour pressure at the actual temperature of the air.

Tables such as the following have been drawn up showing the maximum pressure of aqueous vapour and the mass of vapour present in a given volume of air for various temperatures. It will be seen that the numbers in the two columns (under f and m) are very nearly proportional to each other. They are not quite proportional, for unsaturated vapours only approximately obey the gaseous laws. The proportionality is, however, sufficiently close for all practical purposes.

128. Table showing pressure of aqueous vapour (f) in mm. of mercury, and mass of water (m) in grammes contained in 1 cubic metre of air, with dew-point t.—(*Regnault and Magnus.*)

t	f	m	t	f	m	t	f	m	t	f	m
C.	mm.	gms.	C.	mm.	gms.	C.	mm.	gms	C.	mm.	gms.
−10°	2·0	2·1	0°	4·6	4·9	10°	9·1	9·4	20°	17·4	17·2
− 9	2·2	2·4	1	4·9	5·2	11	9·8	10·0	21	18·5	18·2
− 8	2·4	2·7	2	5·3	5·6	12	10·4	10·6	22	19·7	19·3
− 7	2·6	3·0	3	5·7	6·0	13	11·1	11·3	23	20·9	20·4
− 6	2·8	3·2	4	6·1	6·4	14	11·9	12·0	24	22·2	21·5
− 5	3·1	3·5	5	6·5	6·8	15	12·7	12·8	25	23·6	22·9
− 4	3·3	3·8	6	7·0	7·3	16	13·5	13·6	26	25·0	24·2
− 3	3·6	4·1	7	7·5	7·7	17	14·4	14·5	27	26·5	25·6
− 2	3·9	4·4	8	8·0	8·1	18	15·4	15·1	28	28·1	27·0
− 1	4·2	4·6	9	8·5	8·8	19	16·3	16·2	29	29·8	28·6
− 0	4·6	4·9	10	9·1	9·4	20	17·4	17·2	30	31·6	30·1

Example. *To find the relative humidity of the air, given the actual temperature and the dew-point.*

Suppose the temperature of the air is 15° C. and the dew-point is 10° C. The tables show that the maximum pressure at 15° C. is 12·7, and at 10° C. is 9·1. Therefore the relative humidity is 9·1/12·7 = ·721 or 72 per cent.

129. Dew-point hygrometers. These are instruments devised for determining the dew-point, and thus, as explained above, indicating the hygrometric state of the air; they assume somewhat different forms, but the principle of action is the same in each. The more important forms are given below:—

130. Regnault's hygrometer. This consists (Fig. 81) of a glass tube T, closed below by a thin silver cap S. Its mouth is fitted with a cork carrying a thermometer t and a piece of tubing d, *both passing down nearly to the bottom of the tube.* To use the instrument ether is placed in the tube, and a current of air is drawn through by the aspirator

A;* entering by *d*, the air bubbles through the liquid and enters A after passing through the side tube *e* and the vertical tube V. This passage of air through the ether in

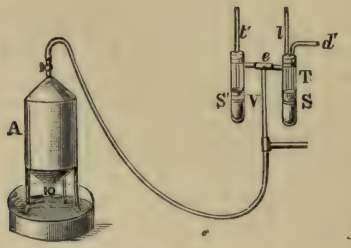


Fig. 81.

S causes rapid evaporation to take place; the temperature of the liquid therefore falls, and ultimately the aqueous vapour in the air condenses on the surface of S. The temperature is then read. This will be below the dew-point, as no dew is seen till a perceptible quantity has formed. The aspirator is now shut off; the temperature of S rises, and when the last trace of dew vanishes the temperature is again read. The mean of the readings is the dew-point.

By regulating the current of air the cooling of the liquid in S may be made sufficiently slow to note, with accuracy, the temperature at which the film of moisture commences to form on S. This observation is usually further facilitated by having a second tube S′, similar to the hygrometer tube, supported on the same stand, in such a position that the appearance of the two silver caps may be easily contrasted, and the dimness due to the deposition of moisture on S more readily detected. S′ is not, however, an essential part of the instrument: it has no connection with S or V,

* The aspirator should in practice be placed a considerable distance away from the hygrometer, as if close the water in the pan alters the state of the air near the hygrometer.

and serves only for comparison, as explained above, and to carry a thermometer t', which gives the temperature of the air.

In order to obtain good determinations with any form of dew-point hygrometer, the experiment should be repeated until the temperatures of appearance and disappearance of the film of moisture on the cold surface are very nearly the same, say within ·5° of each other. The observations should be made by means of a telescope at some distance from the apparatus, so that all risk of error, due to the proximity of the observer, is obviated. If a telescope is not available the observer should place a large sheet of glass between himself and the instrument. If a fine wire is drawn over the surface of the bulb it is easy to detect the first traces of moisture.

131. Dines's hygrometer. This is a simple and convenient form of hygrometer (Fig. 82). R is a small reservoir which communicates, by means of a tube *t*, with the lower compartment of a small double chamber C; in the upper compartment is placed the bulb of a thermometer. The stem of the thermometer is visible from above through a narrow groove cut in the stand, and the chamber C is roofed over by a piece of thin black glass *gg*, on which the slightest film of moisture is easily perceived. To determine the dew-point cold water, containing ice if necessary, is put in R and a slow stream allowed to flow through C, around the thermometer, under the black glass, and out at *d*. The glass is thus cooled, and when its temperature reaches the dew-point it becomes dimmed with dew. The flow is then stopped and the temperature, when the dimness just vanishes, is read. The experiment is then repeated as with Regnault's hygrometer. In another form of Dines's instrument two reservoirs are used, one containing cold water, the other water at the air temperature. Better results are thus obtained.

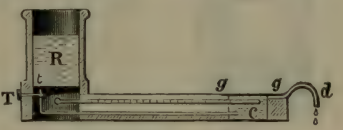


Fig. 82.

132. Wet and dry bulb hygrometer. This instrument is sometimes called *Mason's hygrometer*. It consists of two exactly similar thermometers, mounted on the same stand in such a way that the air can circulate freely round their bulbs. One thermometer is used merely to give the temperature of the air; the other has its bulb covered with muslin, which is kept moist by means of an attached wick communicating with a vessel of water. The evaporation which takes place from this wet bulb produces a fall of temperature, and consequently the wet bulb thermometer always reads lower than the dry bulb. Now it is evident that this difference of temperature between the two thermometers, depending, as it does, on the cold produced by evaporation from the wet bulb, will be proportional to the rate at which evaporation takes place, and, as this depends on the quantity of moisture already in the air, we can see that the difference of temperature of the thermometers is indirectly related to the hygrometric state of the air. If this relation can be discovered, it may serve to determine the humidity of the air, but, as the assumptions involved are somewhat doubtful, it is usual to determine the dew-point from the thermometric readings, by means of tables, empirically constructed from the simultaneous indications of a dew-point instrument.

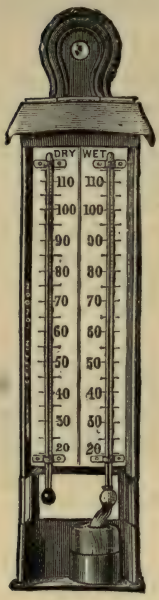


Fig. 83.

133. Hygroscopes are contrivances with hair, catgut, etc., which serve to indicate the humidity of the air by means of some effect produced by the absorption of moisture. Many substances absorb moisture to a degree dependent on the quantity present in the air, and this absorption is generally accompanied by some variation in the form or

condition of the substance. The nature of the variation depends on the structure; for instance, membranous bodies, such as paper, parchment, etc., expand and contract according as they absorb or lose moisture; filamentous substances change much more in thickness than in length; ropes, strings, and threads, which are composed of twisted fibres, swell out and shorten * on the absorption of moisture. Hair, owing to its peculiar structure, shortens as the humidity increases. A single hair is made up of a series of cones fitting one into the other; on absorption of moisture, it swells out, and the vertical angles of the cones increase. Consequently the cones become flatter and the length of the hair decreases.

De Saussure's hygroscope (Fig. 84) is of historic interest, as being the first instrument devised for hygrometric purposes. It consists of a hair from which all grease has been removed by washing it in a solution of soda. It is fastened at one end to the screw-pin *s*, and at the other passes once round the pulley *p*, and carries the small weight *w*, which keeps it tightly stretched. Attached to the pulley is the needle *n*, which indicates, on a magnified scale, the change in length of the hair corresponding to changes in the humidity of the air. The scale round which the needle moves is graduated to indicate various degrees of humidity, but the indications are only relative, and the instrument is never used for scientific purposes.

Fig. 84.

In some parts of the country long strips of seaweed are kept as weather indicators. On the approach of wet weather the salt in the weed absorbs moisture and the weed becomes wet and slimy. "Corns" which grow on

* It should be noticed that the individual fibres of a rope, etc., probably increase *slightly* in length, but the rope, as a whole, shortens for the same reason that a number of thick strings form, on twisting, a shorter rope than the same number of fine strings of the same length twisted to the same extent.

the feet of some people also serve as efficient weather indicators. It may also be mentioned at this point that meteorological observations show that there is no connection between the moon and the weather.

134. The mass of a given volume of moist air. To calculate this we require the following two facts.

(1) The mass of one litre of dry air at 0° C. and a pressure of 760 mm. of mercury is 1·293 grammes.

(2) The mass of one litre of aqueous vapour at any temperature and pressure at which it exists is $\frac{5}{8}$ of the mass of one litre of dry air at the same temperature and pressure.

Consider a volume of V litres of moist air at pressure P and temperature $t°$ C. and let f denote the pressure exerted by the aqueous vapour present. The entire gaseous mass may be considered as made up of two parts:

(a) A volume V of dry air at pressure $(P - f)$ and temperature $t°$ C., the mass of which is given by

$$M' = V \times 1{\cdot}293 \times \frac{273}{273 + t} \times \frac{P - f}{760}.$$

(b) A volume V of aqueous vapour, at pressure f and temperature $t°$ C., the mass of which is given by

$$m = \tfrac{5}{8}\left(V \times 1{\cdot}293 \times \frac{273}{273 + t} \times \frac{f}{760}\right).$$

Hence the total mass of the volume V of moist air is given by

$$\begin{aligned} M &= M' + m \\ &= \frac{V \times 1{\cdot}293 \times 273}{(273 + t) \times 760}\{(P - f) + \tfrac{5}{8}f\} \\ &= \frac{V \times 1{\cdot}293 \times 273}{(273 + t) \times 760}(P - \tfrac{3}{8}f). \end{aligned}$$

135. Formation of cloud. Clouds are formed by local condensation of the water vapour present in the air in the upper regions of the atmosphere. They consist probably of very minute drops of water, which, on account of their small size, fall very slowly through the air.

The formation of a cloud is illustrated by what takes place in **Exp. 86.** The jet of steam issuing from the flask supplies water vapour more rapidly than it can be distributed throughout the air by diffusion. Hence, in the region of over-supply, the vapour condenses and forms a small "cloud." It should be specially noted that this

cloud really marks a region of condensation which is fed by vapour from the flask, and is limited in extent by continuous vaporisation and diffusion from its outer surface. The cloud is therefore not a floating mass of unchanging material, but a constantly changing mass of condensed vapour, which is continuously formed by condensation in the condensation region, and is as rapidly dissipated by vaporisation and diffusion at its outer surface. A cloud cannot therefore be said to float; it is merely a formation marking any region in which the conditions which determine condensation may obtain. The great variety of cloud-forms is due to the variety of the conditions under which they are formed. A typical case of cloud formation is that in which a great column of heated air laden with water vapour ascends into the upper air. The flow of warm moist air into the colder upper air determines, just as in **Exp. 86**, a region of condensation, due to over-supply of vapour, and huge clouds known as *cumulus* clouds form at the top of the ascending column. Similarly, when currents of air at different temperatures meet, the layers of contact may become regions of condensation and clouds, often of the *stratus* type, may form in these regions.

In the case of clouds formed at great heights, where the temperature is very low, the constituent drops may be frozen into small ice particles, or the vapour may be condensed in minute ice crystals, forming the beautiful clouds known as *cirrus* clouds. The dark rain clouds of cumulus-like structure are called *nimbus* clouds.

If the lower layers of air are saturated the small cloud particles collect into drops and fall as **rain**. If the cold is sufficiently intense to freeze the minute particles before they collect into rain-drops a fall of snow results. If the rain-drops form and are then frozen the result is **hail**.

136. Formation of fog or mist. When the vapour in the air condenses in regions at or near the surface of the earth *fog* or *mist* is produced. The mode of formation of fog is practically the same as in the case of a cloud, the difference in the appearance of the phenomena being mainly due to the fact that clouds are formed at considerable heights

above the observer, while fogs and mists are formed near the surface of the earth. In the case of fog, however, the masses of air involved are often at rest or in very slow motion, and the bank of fog produced is more stationary and fixed in form than a cloud usually is.

The most frequent cause of fog is the cooling of the air below the dew-point. The fog region near Newfoundland, for example, is due to the cooling of the warm vapour-laden wind from over the Gulf Stream by the cold air of the Labrador current. Similarly, when warm saturated air comes in contact with a mountain top the cold surface of the mountain cools the air below the dew-point and a thick mist envelops the mountain.

Fogs often form in river valleys and over lakes. If the water is warmer than the air a fog is formed by the over-supply of vapour from its surface. If, however, the water is colder than the air the layers of air in contact with it are cooled below the dew-point, and a fog or mist appears. As explained above, the formation of fog is greatly facilitated by the presence of small particles of matter in the air. It seems probable, indeed, that nuclei of some kind are essential as centres of condensation for the formation of a fog. This explains the frequency of dense fogs in large towns. The smoke and dust particles in the air serve as convenient centres of condensation, so that directly the air is cooled below the dew-point, a fog composed of comparatively large opaque drops of water with a core of soot or dust is produced.

137. Formation of dew.* A satisfactory theory of the formation of dew was first given by Wells. During the day objects on the surface of the earth are heated by direct radiation, and the air in contact with them is also heated, and generally contains a large quantity of aqueous vapour. During the night cooling takes place, and substances which radiate well quickly cool below the temperature of the surrounding air. As a consequence the air in contact with

* This cannot be fully understood until the chapter on Radiation has been read.

these bodies also cools, and ultimately the *dew-point* (Art. **126**) is reached, at which the air is saturated with the vapour it contains, and if further cooling takes place a portion of the vapour is deposited as dew on the surfaces of the adjacent bodies.

On this theory it is evident that the following conditions are necessary for a copious deposition of dew:—

1. A clear sky, to allow free radiation into space.

2. A calm state of the atmosphere, in order that the air in contact with any object may remain in position until cooled below the dew-point.

3. The objects on which dew is deposited must be—

(*a*) Good radiators, in order that they may cool rapidly.

(*b*) Bad conductors, in order that the loss of heat by radiation may not be counterbalanced by a gain of heat from the earth by conduction.

(*c*) Placed near the earth. When the substance is some distance above the earth the air in contact with it, on being cooled, sinks towards the earth, and its place is supplied by warmer air from above. In this way none of the air which comes in contact with the body may be sufficiently cooled to deposit its vapour as dew.

Wells' theory has been confirmed by recent investigation, but other experiments have shown that dew is derived not only from vapour present in the air, but also from vapour arising from the earth and from the vegetation on which the dew is formed.

EXERCISES XIII.

1. If a glass of cold water is brought into a hot room, the outside of the glass frequently becomes wet, though none of the water is spilt. Explain this.

2. Why does "steam" rise from a kettle or basin of boiling water, though under ordinary atmospheric conditions cloud or mist tends to fall slowly?

3. Why will spectacles often become dim on a frosty day if the wearer goes direct from the open air into a warm room? Why does the dimness soon pass away?

4. What becomes of the steam which a boiling kettle discharges into a room?

5. On a hot summer day, immediately after a rain-storm, a block of ice on a cart appears to be "steaming." Explain this phenomenon.

6. Define dew-point. How is dew formed, and why is it more copious on some substances than others?

7. Under what circumstances is dew formed? What is the dew-point? How will the determination of the dew-point give the pressure of the vapour of water in the atmosphere?

8. How would you determine the hygrometric state of the air by means of Regnault's hygrometer?

9. What circumstance determines whether a towel exposed to the air shall dry or shall become damp? Why does a damp cloth exposed to draught become very cold?

10. Describe any "dew-point hygrometer," and explain how by its means the moisture of the air may be determined.

11. I wrote yesterday for some time in a room with a temperature of 60° F. I passed from it into another room with a temperature of 90° F. The spectacles which I wore were immediately dimmed in the warm room; but after a few minutes the dimness disappeared. Explain all this.

12. A mist is often said to "rise from" the ground. Explain in what sense this phrase is incorrect.

13. Explain the phenomena of snow, hail, and hoar-frost.

14. How is the reading of a thermometer altered by wrapping a wet rag around the bulb? What will happen if the rag is wetted with (1) ether, (2) oil instead of water? How do you explain the various results?

15. Two cubic metres of air at 17° C. are drawn through an absorption hygrometer, and 24·12 grammes of water were deposited in the tubes. Find the relative humidity of the air.

16. Find the hygrometric state of the air at 20° C., the dew-point being 10° C.

17. Find the dew-point when the relative humidity at 26·4° C. is 20 per cent.

18. Find the mass of a litre of moist air at 18° C., the dew-point being 5° C. and the barometric height 757·5 mm.

19. The relative humidity of air at 16° C., expressed as a percentage, is 22·5; find the dew-point.

20. Two cubic metres of air, at 14° C., are found to contain 18·56 grammes of moisture. Find the dew-point and relative humidity of the air.

21. Twenty litres of moist air, at 15° C., are drawn through an absorption hygrometer, and found to contain 0·1863 gramme of moisture. What is the hygrometric state of the air?

22. Show the importance in the economy of Nature of the following properties of water: (1) its great specific heat, (2) its great latent heat of fusion, (3) its great latent heat of vaporisation, (4) its expansion on freezing, (5) its greatest density occurring some distance above its freezing point.

EXAMINATION QUESTIONS.

1. What is *vapour pressure?*
A long barometer tube can be moved up and down in a deep vessel nearly filled with mercury. The reading of the barometer is 75 cms. A few drops of water are introduced into the barometer tube, and the height of the mercury column falls to 73·5 cms., a film of moisture being visible on the top of the column. Explain the fall and state, giving reasons, whether the height of the barometer column or the amount of moisture visible would alter if the tube were gradually (1) raised, (2) lowered, in the vessel.

2. What is the difference between a saturated and an unsaturated vapour? What is the way to ascertain whether the vapour with which a given vessel is filled is saturated or unsaturated?

3. What is a saturated vapour?
A steel vessel has carbonic acid gas pumped into it at a steady rate, the vessel being kept constantly at a temperature of 15° C. State in general terms what you would expect the indications of a pressure gauge attached to the vessel to be during the process of filling.

4. Describe how you would measure the pressure of saturated water vapour at different temperatures between 0° C. and 100° C.
Describe also how you would prove that, when excess of water is introduced into a vacuum, it produces the same increase of pressure that it would produce if the space were occupied by dry air.

5. How would you experimentally determine the vapour tension of alcohol at different temperatures?

6. Describe carefully the phenomenon of boiling. What is the distinction between boiling and ordinary evaporation? On what circumstances does the boiling point of a liquid depend?

7. Water is heated over a flame in a wide shallow dish with a loosely-fitting lid. Describe in detail what happens inside the dish.

Would absence of the lid produce any effect upon the process (1) if the flame were small, (2) if it were very large?

8. How would you carry out an experiment to show that the vapour-pressure of a liquid at its boiling point is equal to the atmospheric pressure?

Give some explanation of this fact.

9. Describe an experiment to show that the boiling point of water is raised by increase of atmospheric pressure over its surface. What would be the immediate, and what would be the ultimate, result if water at 20° C. were introduced into a perfectly maintained vacuum?

10. A quantity of dry air is confined in a flask provided with a pressure gauge. The pressure is initially 76 cms. of mercury. How would the pressure change if water were continually introduced a drop at a time into the flask, if the temperature of the flask and its contents were maintained (i) at 15° C., (ii) at 100° C.? Describe the details of the apparatus by which the above experiments could be carried out.

11. What is meant by the Latent Heat of steam? Describe an experiment by which you might determine it, supplying possible data, and working out the result from them.

12. Explain the statement that the latent heats of vaporisation of water and of liquefaction of ice are 536 and 80 units respectively.

A certain quantity of water at 0° C. is raised to the boiling point by condensing in it 5 grammes of steam at 100° C. The temperature of the whole is then reduced to the freezing point by adding ice at 0°. Find the amount of ice required.

13. The specific heat of a certain liquid is 0·42 (calories per gramme per degree centigrade), and its latent heat of vaporisation at its boiling point, 60° C., is 350 (calories per gramme). Calculate how much steam must be condensed in a tube leading through it to warm 2 kilogrammes of the liquid from the air temperature 15° C. to the boiling point of the liquid, and to boil away 20 grammes of the liquid. The condensed water remains in the tube. The thermal capacities of the containing vessel and condensing tube may be neglected.

14. A copper sphere is suspended by a fine wire in a chamber whose temperature is 20° C. Steam at 100° C. is then blown through the chamber until the temperature of the chamber and steam is steady at 100° C. Explain carefully why the sphere becomes coated with a definite quantity of moisture. If the steam is cut off before this steady temperature is attained, the highest common temperature of the sphere and moisture being 70° C., calculate the quantity of moisture on the sphere, the mass of the sphere being 100 grammes, the specific heat of copper 0·1, and the latent heat of vaporisation of water at 100° C. 540 calories per gramme.

15. What is meant by the *water equivalent* of a calorimeter?
A given metal and a given liquid have specific heats of 0·2 and 0·6 respectively. If the boiling point of the liquid is 80° C. and its latent heat of vaporisation 200 calories per gramme, what will be the result of mixing 100 grammes of the powdered metal at 220° C. with 50 grammes of the liquid at 20° C.?

16. Heat is applied at a uniform rate to a pound of ice at −10° C. until it becomes steam at 120° C., the pressure remaining normal throughout the process. Describe, best by the aid of curves, the way in which temperature and volume vary with the time from the start.

17. Define the terms *hygrometric state* and *dew-point*. How may the hygrometric state be inferred from the dew-point?

18. How is the pressure of the aqueous vapour in the atmosphere connected with the dew-point? Describe a method of determining the dew-point.

19. How is dew formed?
Give two conditions favourable, and two conditions unfavourable, to its formation.

20. What experiments would you perform to illustrate the mode of formation of dew and of mist respectively?
On a clear still evening, after a wet summer's day, mist seems to rise from a grass lawn. Explain carefully how the mist forms.
Would covering the grass with canvas prevent the formation of mist? Give reasons.

21. You are given a barometer, an instrument for determining the dew-point, and a table showing the pressure of saturated water vapour at different temperatures. How would you estimate the pressure of the dry air, and that of the water vapour in the atmosphere?

22. On a cold frosty night it often happens that the bottom of a valley is colder than the top of a neighbouring hill. Explain this.

CHAPTER XIV.

CONDUCTION OF HEAT.

138. Three modes of transference of heat. Heat may be transferred from one point to another in three different ways—*conduction, convection, radiation.* In *conduction* heat imparted to any part of a body sets the molecules of that part in more energetic vibration. These transmit the vibration to their neighbours, and thus the heat travels slowly along. In *convection,* which is only possible in gases and liquids, the parts heated expand, and, becoming less dense, rise and carry their heat *by their own motion* to other parts. In *radiation* the hot body imparts its vibration to the ether (see Art. **177**), in which waves are set up, which travel in straight lines in all directions with enormous velocity, and through vacuous spaces even more freely than through air or other matter.

139. Conductivity. The fact that heat may be transmitted or conducted through the substance of a body is familiar from every-day experience. When the end of a poker is placed in the fire the heating effect is not confined to the portion actually in the fire. The portion outside the fire is also heated in a way which indicates that heat must be conducted through the substance of the poker from the end in the fire towards the other end. When boiling water is poured into a metal vessel the *outside* of the vessel quickly becomes hot, showing that heat has been conducted through the thickness of the vessel. When a

fire is lighted in a closed range the portions of the range not in direct contact with the fire become heated by the heat conducted from the fire through the iron of the range. If a flat-iron or block of any substance be placed on a hot-stove plate it becomes heated throughout its mass by the conduction of heat from its under surface.

The power which a substance possesses of allowing heat to be conducted through it is known as the **conductivity** of the substance.

140. Substances differ in conductivity. Substances differ greatly in their power of conducting heat. A rod of wood if placed with one end in the fire gives little indication of the conduction of heat outwards from the end in the fire. If boiling water is poured into a thick wood vessel the heating of the outer surface is very slight. It also takes place very slowly. The difference in the heat sensations experienced on touching different substances is also, as explained in Chapter I., due to difference in the conductivity of the substances. For example, in the hot room of a Turkish bath where all the objects are at the same temperature, metallic objects feel much hotter than others and may inflict a burn. Similarly in a cold room metallic objects feel very cold to the touch, while wooden objects, carpets, clothing, and other similar objects give no pronounced sensation of cold, and yet all the objects in the room may be shown to have the same temperature by bringing them successively in contact with a thermometer. This difference of sensation is obviously due to a difference in the conductivity of the substances touched; good conducting substances such as metals rapidly conduct heat to or from the hand and thus cause a more marked sensation of heat or cold.

The following experiments further illustrate the difference in the conductivity of different substances:—

Exp. 88. Fit the end of a short wooden cylinder into a thick brass tube of the same external diameter. Wrap a piece of *thin* paper round the cylinder at the junction of the wood and brass and hold the paper in the flame of a spirit lamp or Bunsen burner.

It will be found that the paper over the wood is badly scorched, while that over the brass is not scorched. This is due to the fact that the brass, being a good conductor, conducts the heat away from the paper sufficiently rapidly to prevent scorching, whereas the wood being a bad conductor is unable to do this.

Exp. 89. Repeat **Exp. 88**, using *thick* paper. It will be found that the paper is scorched wherever the flame touched it. The low conductivity of the paper prevents the heat from being transmitted through its thickness, and the presence of the good conducting brass beneath has therefore no effect on the result.

Exp. 90. Take similar strips of silver, copper, and iron and place them with one end in hot water in a beaker. The other end of the silver strip soon becomes very hot, the copper also gets hot, but not to the same degree as the silver, while the iron strip is heated to much less extent than either the silver or the copper.

Exp. 91. Twist an iron and a copper wire together at one end. At four or five inches from the joint fasten a small marble on each wire with beeswax. Heat the joint in a Bunsen flame. Heat is conducted from the joint outwards along each wire and alternately the temperatures of the wires at the points where the marbles are attached will be raised to the melting point of the beeswax and the marbles will drop off. Notice that the marble on the copper wire is the first to fall, showing (in this case) that the conductivity of copper is higher than that of iron. (See **Exp. 98.**)

In general metals are good conductors of heat. Organic substances such as wood, bone, felt, are bad conductors. Brick is also a feeble conductor, while stone, marble, slate, glass, though better conductors than brick and organic substances, are of much lower conductivity than metals.

The fact that some substances have a low conducting power has many familiar applications. Handles of wood or other badly conducting material are usually provided to tools, hot water vessels, and other appliances where the hand has to be protected from heat which might otherwise reach it by conduction. In the case of tea-pots, coffee-pots, kettles, etc., the handle is sometimes of metal, but in that case the handle is separated from the body of the vessel by discs of china or other bad conducting substance.

Exp. 92. A kettle that has been long in use has a thick coating of carbon on the bottom. To show that this coating is a bad conductor, boil water in the kettle, and then support the kettle on the palm of the hand. The kettle and water feel only moderately hot.

The efficacy of clothes and bed clothing is due to their low conductivity. The heat developed in the body is prevented from escaping by the covering of badly conducting material and thus the feeling of warmth is maintained.

A good example of this retention of heat by a hot body surrounded by a bad conductor of heat is found in what is called a Norwegian cooking box, which consists of a wooden box with a thick lining of felt so arranged as to leave a central cylindrical space into which the vessel containing the food fits. The food is partly cooked over the fire in this vessel and while still hot is placed in the box. The lid, also felt-lined, is then lightly closed over it. The heat is so well maintained that the cooking is completed in the box, and after an interval of some hours the temperature will have fallen only a few degrees.

Bad conductors are as effective in keeping a body cool as in keeping it warm. Thus a covering of felt or flannel which will keep a vessel of hot water warm will also keep a piece of ice from melting. In the one case heat is prevented from passing out from the water and in the other from passing into the ice.

Exp. 93. Place some hot water in a metal vessel and note its rate of cooling when placed on a large metal plate without any protection from loss of heat by conduction. Then repeat the observations when the vessel is protected by a covering of felt or by being supported on cork or suspended inside another vessel. Compare and explain the result.

141. The principle of the Davy lamp. When we apply the flame of a lighted match to gas issuing from a jet the temperature of the gas is raised above its temperature of ignition and combustion commences and continues so long as a supply of oxygen is obtainable and the temperature remains above the ignition temperature of the gas.

If, however, a piece of good conducting material be placed in a flame it may conduct away the heat so rapidly that

the temperature falls below the temperature of ignition and combustion ceases. This effect is illustrated by the following experiments.

Exp. 94. Take a piece of thick copper wire and coil it into a spiral of about half a centimetre internal diameter. Light a candle, and when the flame is burning strongly lower the coil of copper wire on to the wick of the candle. The flame at once goes out.

Exp. 95. Take a piece of clean copper gauze, not too fine in mesh, and lower it on to the flame of a Bunsen burner or spirit lamp. It will be found that the flame does not get through the gauze. As the gauze is lowered the flame is kept below it and may be extinguished completely by lowering the gauze on to the burner or wick. The gauze conducts the heat away from the portion of the flame in contact with it sufficiently rapidly to lower the temperature below that at which combustion takes place, and the region of combustion is therefore unable to extend above the gauze.

If the gauze is held over the flame until it is heated to the temperature of ignition it ceases to be effective for the purpose of this experiment.

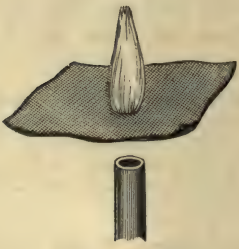


Fig. 85.

Fig. 86. *

Exp. 96. Hold a sheet of gauze about an inch above a Bunsen burner and light the flame *above* the gauze. It will now be found that the flame appears only above the gauze and is prevented by the conductivity of the gauze from extending below it (see Fig. 85).

* Block kindly lent by J. J. Griffin and Sons Ltd., Kingsway, W.C.

The principle illustrated by these experiments has been utilised in the construction of the Davy lamp (Fig. 86). The flame of the lamp is enclosed in a cylinder of wire gauze so that when the lamp is placed in an explosive atmosphere the gases which penetrate to the flame are ignited and burn *inside* the gauze with a peculiar flickering, but the flame produced is unable to extend outwards through the gauze unless the gauze becomes hot. The explosion which would be produced by a naked light is thus averted. As soon as the miner notices the presence of the ghostly fire-damp flame he leaves the working at once and gives notice to his foreman, whereupon the mine is thoroughly ventilated.

It has been found, however, that the Davy lamp is not, in all cases, an efficient protection against explosions in mines.

142. The stationary and variable states during conduction. Imagine a bar placed with one end in the fire, and let us consider what takes place as the heat is transmitted outwards from the fire, along its length. The portion in the fire rapidly increases in temperature and finally takes the temperature of the fire; meanwhile, as the temperature of this portion rises the transverse layer adjacent to it receives heat by conduction, absorbs part of this heat to increase its own temperature, loses a small portion by radiation from its surface and passes on the rest to the adjacent layer, where the process is repeated. When this has gone on for some time a state will ultimately be reached when each layer attains a stationary temperature and ceases to absorb any of the heat passed on to it by the adjacent layer nearer the fire. This state has been called the **stationary state**, and the passage of heat along the bar, when this state is attained, depends, for given conditions, on the conductivity of the material of the bar. The stage previous to the attainment of this stationary or permanent state is known as the **variable state**, for, while it lasts, each layer absorbs some portion of the heat it receives, and is consequently rising in temperature.

During the stationary state each layer passes on to the next *less* heat than it receives by the amount which it loses at its outer surface. Hence, if there is any considerable loss of heat from the outer surface, the quantity of heat that passes from layer to layer rapidly decreases, and, at a comparatively short distance from the source of heat, the flow of heat along the bar becomes negligibly small. It is evident therefore that in comparing the conductivities of different substances by comparing the flow of heat along similar bars of these substances not only must the bars be similar in shape and dimensions, but the nature of the outer surface must also be exactly the same for each bar.

143. Gradient of temperature. In all cases of conduction the rate of transmission of heat, that is, the quantity of heat transmitted per second, in any direction, is found to depend upon the fall of temperature per unit distance in that direction. This *fall of temperature per unit distance* for any direction is called the *gradient of temperature* for that direction. The gradient of temperature for any direction may vary from point to point along that direction, and it is measured at any point by the average gradient over a very short distance taken at the point. For example, in the case of a bar heated at one end the gradient of temperature along the length of the bar decreases from point to point as the distance from the hot end increases. Also, if in a certain portion of the bar there is a fall of 3° C. in 2 mm. the gradient of temperature at the middle of this length of 2 mm. is 15° C. per cm.

144. Influence of specific heat during conduction. The rate at which heat is conducted through any substance, during the *variable state* of the process, depends not only on the conductivity of the substance, but also on its specific heat. If the specific heat of a substance is low the temperatures of the successive layers rise quickly to the stationary points, even when the conductivity is comparatively low, for only a small portion of the heat that travels along is required to heat the substance. Similarly, if the specific heat is high the temperatures of the successive

layers may rise slowly to the stationary values, even when the conductivity of the substance is fairly high.

This question is illustrated by the following experiments:

Exp. 97. Place two bars of different metal, but of the same size and shape, and having an exactly similar surface, end to end, as shown in Fig. 87. To the under surfaces attach with wax small wooden balls, at equal distances along the length of the bars. and heat the bars at their contiguous ends. As the heat is transmitted along their length, the wax melts and the balls drop off in succession. Now the first ball will drop off directly the temperature of the point at which it is attached becomes equal to the melting point of wax, and, provided the distance of this point from the source of heat be not too great, the time required for this to take place depends not only on the conductivity, but also on the specific heat of the material of the bar; because, the lower the specific heat of this material, the greater will be the rise of temperature produced in a given mass of it by the heat supplied in a given time. For this reason the balls may begin to drop off first from the bar which has the lower specific heat, but the greater number of balls will ultimately drop off from the bar which has the greater conductivity; because, the greater the flow of heat along the bar, the further will the rise of temperature necessary to melt the wax be transmitted.

Fig. 87.

Exp. 98. Take two small cylinders, of equal length, one of iron and the other of bismuth, and, after having coated one end of each with wax, place the other end on a hot copper plate. If the cylinder be not too long, and the temperature of the plate be sufficiently high, the wax on the upper end of the bismuth melts first, although the conductivity of iron is more than six times that of bismuth.

This is explained by the fact that the thermal capacity of the bismuth is much less than that of the iron, and consequently it requires more heat to raise the temperature of the iron, through a given range of temperature, than it does to raise the bismuth through the same range; hence, although the heat transmitted along the bismuth is less than that along the iron, the rise of temperature produced in the former is greater. When, however, the stationary state is attained, a greater quantity of heat will be transmitted through the iron than through the bismuth, and the temperature of the upper end of the iron cylinder will ultimately be the greater.

145. Definition of thermal conductivity. Imagine a large plate or wall of any substance, with parallel faces which are maintained at constant temperatures differing by a definite known amount, and consider the conduction of heat during the stationary state through the thickness of this wall from one face to the other. Near the edge of the plate some heat flows laterally outwards and escapes from the edge before passing through the plate, but if we take a small portion of the plate near the centre and at a distance from the edge, it may be assumed that all the heat which enters over any small area on the hot face of the plate passes straight through, *without lateral loss*, and emerges from the corresponding equal area on the cold face.

Under these conditions the quantity of heat which crosses any area of the plate depends upon four things, viz.—

1. The substance of the plate.
2. The area across which the flow of heat takes place.
3. The *gradient of temperature* in the plate, that is, the *difference of temperature per unit thickness* of the plate.
4. The time for which the flow of heat is taken.

It is found that, for a given substance, the flow of heat through the plate is directly proportional to—

1. The area across which the flow takes place.
2. The gradient of temperature in the plate.
3. The time of flow.

The quantity of heat, therefore, which flows in *unit time* through *unit area* of a plate in which there is *a gradient of temperature of one degree per unit thickness* depends only on the substance of the plate and may be taken as a measure of the *thermal conductivity* of the substance.

The thermal conductivity of a substance may therefore be defined as measured by the quantity of heat which flows in unit time across unit area of a plate of unit thickness having one degree difference between the temperatures of its faces.

Thermal conductivity defined this way is sometimes called the *absolute thermal conductivity* of the substance.

146. Formula involving the definition of absolute thermal conductivity. For the purpose of calculation it is convenient to put the statements given above in algebraical form.

Let A denote the area, θ the *difference* between the temperatures of the faces, x the thickness of the plate, and t the time of flow; then, as θ/x is the gradient of temperature for the direction of flow, we have

$$\text{H is directly proportional to A} \,.\, \frac{\theta}{x} \,.\, t,$$

where H denotes the quantity of heat which passes through the area A of the plate in time t. That is,

$$\text{H} = k \,.\, \text{A} \,.\, \frac{\theta}{x} \,.\, t,$$

where k is a constant which measures the **absolute thermal conductivity** of the substance.

It is evident that if $\text{A} = 1$, and $\frac{\theta}{x} = 1$, and $t = 1$, then $\text{H} = k$ as stated in the definition given above.

In applying this relation any *consistent* set of units may be adopted. The system in most general use is the C.G.S. system, where A is in sq. cms., x in cms., t in seconds, θ in degrees C., and H in gramme-degrees C.

Example. It is found that the temperature of the rocks below the Earth's surface increases at the rate of (approximately) one degree C. for each 30 metres of descent. Taking the thermal conductivity of the rocks as ·005 on the C.G.S. system, calculate how much ice would be melted by the heat arriving in the course of a year at a square metre of the Earth's surface from its interior.

Here, the area across which the flow of heat has to be considered is 1 square metre or 10000 sq. cms., the gradient of temperature towards the Earth's surface is 1/30 degree C. per metre or 1/3000 degree C. per cm., and the time for which the flow is required is 1 year or $1 \times 365 \times 24 \times 60 \times 60$ seconds.

Hence we have

Calories of heat arriving per sq. metre of surface per year

$$= \cdot 005 \times 10000 \times \tfrac{1}{3000} \times 365 \times 24 \times 60 \times 60,$$

It requires 80 gramme-degrees to melt 1 gramme of ice, therefore the quantity of ice this quantity of heat will melt is

$$\frac{50 \times 365 \times 24 \times 60 \times 60}{3000 \times 80} \text{ grammes}$$

or

6570 grammes

or a layer of ice 0·7 cm. in thickness.

147. Conductivity of liquids. The conductivity of all liquids, except mercury and other liquid metals, is very low. For this reason it is difficult to exhibit conductivity in a liquid experimentally, and the difficulty is increased by the influence of convection currents and by the conduction of heat through the walls of the containing vessel. If, however, a liquid is heated from *above* convection currents are not produced, and the conduction through the walls of the containing vessel may be made negligibly small by using thin vessels of badly conducting material. The following experiments illustrate the low conductivity of liquids.

Exp. 99. Load a piece of ice by twisting a piece of copper wire round it, and sink it in a long tube nearly full of water (Fig. 88). Fix the tube in an inclined position on a retort stand, and heat it near the top by a spirit lamp. It will be found that the water at the top will boil before much of the ice is melted.

Fig. 88.

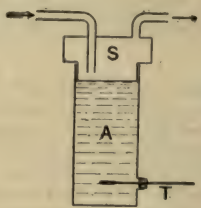


Fig. 89.

Exp. 100. Fit a tall jar, A, made of wood or glass, with a steam-box, *S*, and a thermometer, T, as shown in Fig. 89. Fill the jar with water or other liquid so that the top layer of the liquid is in contact with the lower part of the steam-box. When steam is passed through the box the top layers become heated, and heat should travel downwards through the liquid towards the bulb of the thermometer T.

It will be found that even for comparatively small depths the rise of temperature indicated by the thermometer is very small. If the experiment be performed with mercury in the jar, a very considerable rise of temperature will be observed.

A more elaborate form of this apparatus was first made by Despretz for measuring the conductivity of water.

148. Conductivity of gases. The conductivity of gases is extremely low. Still, the fact that they do conduct is shown by the following experiment.

Exp. 101. Arrange to throw the shadow of a red-hot poker on a screen. Notice how the heating of the air above and below the poker is indicated on the screen. (Explain the indications.) Note that the heating extends but a very little way downwards, showing that air is a bad conductor of heat. In this experiment convection marks the true conductive effect, for as soon as a portion of air below the poker gets hot it becomes less dense and rises.

The very low conductivity of cotton wool, eiderdown, felt, and other fabrics of open texture is due very largely to the air enclosed in the fabric.

149. Experimental determination of absolute conductivity. Theoretically it would appear that the simplest method of determining absolute conductivity of a substance would be to determine the quantity of heat transmitted through a plate of the substance, of given area and thickness, and having a known difference of temperature between its faces, and then to apply the formula of Art. **146**.

This method is often impracticable for two reasons.

(1) It is difficult, if not impossible, to maintain the two faces at constant *known* temperatures.

(2) It is impossible to avoid *lateral* loss of heat by radiation and convection from the edges of the plate.

On this account, the most usual method of investigating conductivity is to take a bar of the substance, the conductivity of which is to be determined, and, keeping one end at a high constant temperature, to note the temperature assumed at different points in its length when the permanent state is attained. From the temperature data thus

obtained, combined with an experimental determination of the loss of heat by radiation and convection from the surface of the bar, the absolute conductivity of the substance can be calculated.* The first important series of experiments based on this method was made by Forbes. To take the temperatures at different points along the rod he bored small holes into the rod, half filled the holes with mercury, and then stood a small mercury thermometer in each hole.

Exp. 102. To find the conductivity of indiarubber. Take a flask fitted up as in Fig. 80 and connect a long piece of ordinary indiarubber tubing to the efflux tube. Fit up a rather large calorimeter, as described in the experiments on calorimetry, put in it a known mass of cold water, a thermometer, and a stirrer. When the water is boiling vigorously immerse a length of the tubing into the water in the calorimeter, keeping the tubing well away from the sides of the calorimeter. Stir gently. The temperature of the water gradually rises: read it every minute. Proceed till the temperature of the water is above that of the surrounding air, and from your readings calculate the rate of rise of temperature when the temperature of the water was equal to that of the air. (This obviates correcting for radiation.) Gently remove the tubing from the water, measure the length, l, of the wetted portion with a millimetre scale, and external diameter, d, of the tubing with a pair of callipers, and the thickness, x, of the walls of the tubing with a screw gauge. The mean diameter of the tubing is $\left(d - \frac{x}{2}\right)$, and therefore the mean area of the indiarubber interposed between the steam and the water is $\pi\left(d - \frac{x}{2}\right)l$. If θ_1 is the rise of temperature of the water in a short time t seconds calculated as indicated above, θ_2 the mean temperature of the water during this interval, and M the mass of water plus the water equivalents of the calorimeter and stirrer, we have by Art. **146**

$$M\theta_1 = k \,.\, \pi\left(d - \frac{x}{2}\right)l \,.\, \frac{(100\text{-}\theta_2)}{x} \,.\, t,$$

from which k can be found. The value of K is about 0·0004 for ordinary rubber tubing. This is so small compared with the value of K for water (·0013) that difficulty (1), above, does not hold, and from the circumstances of the experiment (2), above, scarcely exists either.

---

* See *Higher Text Book of Heat*, Arts. **87**, **88**.

150. Determination of relative conductivity. Rough determinations of the relative conductivity of different substances may be made by means of an apparatus due to Ingenhousz and usually known as *Ingenhousz's apparatus*. This consists of a box (Fig. 90) into which rods of the same thickness, but of different metals, are fixed, as shown in the figure.

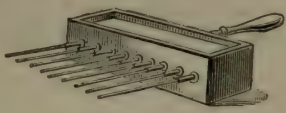


Fig. 90.

The ends of the rods pass into the box, and are raised to a high constant temperature by filling the box with hot water or oil. The portions outside the box are coated with wax, so that, as heat is transmitted along their length, the wax gradually melts.

During an experiment the bath should be kept at a constant temperature by means of a burner, and the rods should be protected from direct heating by means of a screen which is, in some forms of the apparatus, attached to the box so as to screen the rods from the flame.

The front plate of the box should be of thick copper and the rods should be made to fit exactly, without corks, into holes bored in this plate.

Instead of coating the rods with wax it is much more satisfactory to place the wax in a narrow longitudinal groove running the whole length of each rod. The greater part of the surface can then be polished or plated so as to secure exact similarity of surface.

When the permanent state is attained the relative lengths along which the melting takes place depend on the conductivities of the metals employed.

To deduce the relative values of the conductivities from the lengths of wax melted, let us consider two rods A and B, of which the melting on A has proceeded twice as far as that on B. The temperature gradient of A is thus only half that of B. Now when the permanent state is attained all the heat passing through any section of the bar is radiated from the surface of the bar beyond that section. Since the amount of heat radiated from a bar is proportional to the product of the area exposed and the

average difference of temperature between the bar and its surroundings, it follows that the heat radiated from the part of the bar A on which the wax is melted is twice that of the heat radiated from the part of B on which the wax has melted. Substituting in the formula

$$H = k A \frac{\theta}{x} . t$$

of Art. **146**, we get $H_A = k_A A_A \left(\frac{\theta}{x}\right)_A . t$ for bar A

and $H_B = k_B A_B \left(\frac{\theta}{x}\right)_B . t$ for bar B.

Now $A_A = A_B$, for the bars have the same dimensions,

$$\therefore \frac{H_A}{H_B} = \frac{k_A \left(\frac{\theta}{x}\right)_A}{k_B \left(\frac{\theta}{x}\right)_B},$$

i.e.

$$\frac{k_A}{k_B} = \frac{H_A \left(\frac{\theta}{x}\right)_B}{H_B \left(\frac{\theta}{x}\right)_A};$$

but $H_A = 2 H_B$ and $\left(\frac{\theta}{x}\right)_B = 2 \left(\frac{\theta}{x}\right)_A$,

$$\therefore \frac{k_A}{k_B} = \frac{2}{1} \cdot \frac{2}{1} = \frac{4}{1},$$

i.e. the conductivity of the substance of the rod A is four times that of the substance of rod B. The above proof may be made quite general. Thus, if the length of the melted wax on A is n times that of the melted wax on B, we have as before—

$$H_A = n H_B \text{ and } \left(\frac{\theta}{x}\right)_B = n \left(\frac{\theta}{x}\right)_A,$$

$$\therefore \frac{k_A}{k_B} = \frac{n}{1} \cdot \frac{n}{1} = n^2,$$

i.e. **the conductivities are as the squares of the lengths along which the wax has melted.**

Example. In an experiment performed, Ingenhousz's apparatus, the lengths of wax melted on the iron and bismuth rods were 7·7 and 3·0 inches respectively. Compare their conductivities.

By the relation obtained above

$$\frac{\text{Conductivity of Iron}}{\text{Conductivity of Bismuth}} = \frac{(7{\cdot}7)^2}{(3{\cdot}0)^2} = \frac{59{\cdot}3}{9} = 6{\cdot}6.$$

TABLE OF ABSOLUTE THERMAL CONDUCTIVITY.

Silver	1·10	Bismuth.....	0·018
Copper.....	0·72	Water (15°)	0·0014
Brass	0·20	Oak.....	0·0006
Iron	0·15 ~ 0·18	Air	0·00057
Steel.....	0·06 ~ 0·14	Crown glass	0·0005
Lead.....	0·084	Indiarubber	0·0004

EXERCISES XIV.

1. In a room without a fire I place my hand upon the carpet and the fender in succession. The fender is much colder to the touch than the carpet, although when examined by a thermometer they are found to be at the same temperature. Explain the observation.

2. A piece of thin paper is wrapped round a rod which is composed partly of metal and partly of wood. On holding the rod in a spirit flame the paper is charred where it covers the wood, but not where it covers the metal. Explain the cause of this.

3. Explain the use of the wire gauze surrounding the flame in the lamp used in coal-mines.

4. Find the quantity of heat that will be transmitted, in 1 hour, across a plate of copper 1 sq. metre in area and 5 cms. thick, the difference between the temperatures of its faces being 10° C. The absolute conductivity of copper is 0·72.

5. It is found that 6,545,000 gramme-degrees of heat are transmitted, per minute, across a sheet of silver, 100 sq. cms. in area and 1 mm. thick, with a difference between the temperatures of its faces of 100° C. Find, in C.G.S. units, the absolute conductivity of silver.

6. Peclet has stated that the quantity of heat which passes, in an hour, through a plate of lead 1 sq. metre in area and 1 cm. thick, with a difference of 1° C. between the temperature of its surfaces, is 1383 kilogramme-degrees. What value does this give for the absolute conductivity of lead in the C.G.S. system?

7. The absolute conductivity of silver in the C.G.S. units is 1·10; how many heat-units will pass, per minute, across a plate of silver, 1 metre long, 1 metre broad, and 5 cms. thick, when its opposite faces are kept at temperatures differing by 100° C.?

8. The thermal conductivity of felt, in C.G.S. units, is 0·000087; find the quantity of heat that is transmitted, in one hour, through a layer of felt 1 cm. in thickness and 20 sq. cms. in area, when its opposite faces are kept at temperatures differing by 20° C.

9. A square metre of a substance, 1 cm. thick, has one side kept at 100° C., and the other, by means of ice, at 0° C. In the course of 10 minutes one kilogramme of ice is melted by this operation. Calculate the conductivity of the substance, assuming the latent heat of water to be 80.

CHAPTER XV.

CONVECTION.

151. Convection currents. Convection currents are set up in a fluid when the uniformity of density throughout its mass is disturbed by unequal heating. When a mass of liquid, for example, is heated from below the portions first heated expand, and thus becoming less dense than the surrounding liquid, are displaced upwards and give rise to an upward current from the point of heating. The liquid thus displaced upwards is replaced by the cooler liquid adjacent to it, and in this way downward and lateral currents feeding the upward current are established.

Similarly, if a gas flame or other heated body be placed in the middle of a room containing air at a uniform temperature, convection currents are at once set up. The air in the neighbourhood of the flame ascends and is replaced by air supplied by downward currents external to the ascending current.

Fig. 91.

Exp. 103. In a beaker of water strew some paper pulp made by grinding a piece of blotting paper in a mortar with a little water. Place it on a piece of wire gauze on a tripod stand, and when the water has become nearly still put the flame of a Bunsen burner under the middle of the beaker (Fig. 91). Note the ascent of the warm water up the middle of the beaker and the indraught of cold water from the top and sides.

Exp. 104. In a similar beaker of water and paper pulp put a piece of ice. When the commotion due to the introduction of the ice has subsided, note the descending cold current from the melting ice and the indraught currents of warmer water along the top towards the ice.

Exp. 105. Fill a *large* flask with smoke and note the convection currents set up by a small candle flame placed at the centre of the flask.

Exp. 106. Light a simple gas burner in a room where the air is not disturbed by draughts. Hold a piece of smoking paper near the burner and note the upward direction of the convection current. Also find in the same way, if possible, the direction of the currents which feed this upward current.

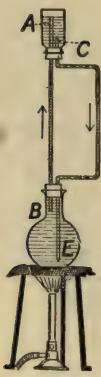


Fig. 92.

Exp. 107. Open a door between a warm room and the cooler passage or hall. Hold a lighted candle in the open doorway near the top and near the bottom and note, from the direction of the flame, the direction of the air current in each position. The current will be found to be outward from the room at the top and inward at the bottom. Close the door and find the direction at the key-hole.

Exp. 108. Set up the apparatus shown in Fig. 92, and fill the flask with litmus solution and the bottle with dilute acid. Heat the flask gently. Note the ascending current of litmus in B A, and the descending current of acid in C D E as it issues at E. Note also that the circulating liquid quickly assumes a uniform red tint, showing the rapid "mixing" effect of the convective circulation.

If acid coloured with litmus is placed in the flask, and alkali coloured with litmus in the bottle, and the quantities be adjusted to give exact neutralisation the experiment is more effective.

152. Distribution of heat by convection. When a liquid in a vessel is heated from below the convection currents set up establish a system of circulation in the liquid. Under this system the liquid circulates round and round, and every portion of it is, in turn, brought near the source of heat. In this way every part of the liquid is rapidly heated by the heat supplied at one point. It must, however, be remembered that although convection currents

thus greatly facilitate the uniform heating of a mass of fluid, yet the transfer of heat from the source to the fluid takes place by *conduction*. Thus convection must be considered as a process which accelerates the uniform distribution of heat throughout a mass of fluid by setting up a condition of things extremely favourable to the transfer of heat by conduction.

Hence, convection in fluids counterbalance, in a great measure, their low conductivity. Thus water may be boiled in a paper vessel because the heat is carried away from the paper, by convection, sufficiently rapidly to prevent the paper being scorched. Similarly, in a mass of gas, uniformity of temperature is established more quickly by convection than, in a mass of good conducting material, by conduction.

When a body cools in air or other gas, the cooling is partly due to radiation, but mainly to convection. The circulation of the gas over the exposed surface carries heat away from the cooling body. The greater the specific heat of the gas and the greater its conductivity the more rapid is the cooling.

Exp. 109. Fit a piece of platinum wire, about 20 cms. long, into a wide glass tube fitted as shown in Fig. 93. Heat the wire to redness by passing a constant electric current through it and note the

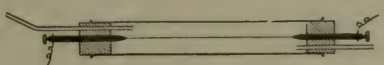


Fig. 93.

cooling effect produced by filling the tube with different gases. It will be found that hydrogen produces the greatest cooling effect.

153. Ventilation. The simpler systems of ventilation are merely methods for establishing convection currents between the outside air and the air in the room to be ventilated in such a way as to promote free circulation of the air without creating draughts. The following experiment illustrates the principles applied in ventilation.

Exp. 110. Stand a candle in a saucer. Pour water around it. Light the candle and stand a tall cylindrical glass tube, such as a lamp chimney, over it. Note the candle goes out. This is due to lack of ventilation ; no air can get in underneath the chimney —the water prevents that—and no air can flow down the chimney.

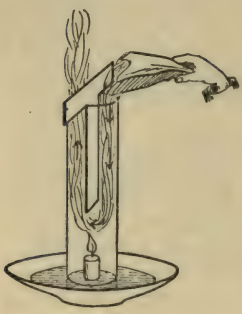


Fig. 94.

Repeat the experiment, this time introducing down the chimney a piece of cardboard or metal shaped liked a T (Fig. 94). Notice that the candle continues to burn. There is a down-draught one side of the T-piece and an up-draught on the other side. To show this, hold some smouldering brown paper near one side of the cardboard and note the direction in which the smoke moves.

The experiment works much better if the candle is shifted so as to be underneath the column in which the up-draught occurs.

The draught in a chimney is also a means of ventilation. The heated air ascends and is replaced by colder air from the room, and thus a fire in a room to which the outer air has access is an effective means of ventilation. Similarly in mines two shafts used to be sunk, and these were connected at the bottom by the workings. A fire was lighted at the bottom of one shaft. A current of fresh air then descended the other and permeated the workings.

154. Winds. The familiar phenomena of winds are good examples of convection currents, set up in the atmosphere by unequal heating. As examples we may consider :—

Land and sea breezes. Land absorbs solar heat more rapidly than water, and, owing to its smaller specific heat, is raised to a higher temperature by the heat thus absorbed. For the same reason land loses heat more rapidly than water. Hence, during the day the land gets heated by solar radiation to a greater degree than the sea, and the air in contact with it also gets heated, expands, and, rising, is replaced by the cooler air over the sea, thus causing a

sea breeze. After sunset, however, the land cools more rapidly than the sea, and ultimately the air over the land becomes colder than that over the sea; the direction of the current of air is then reversed, and during the night a **land breeze** prevails.

The Trade winds. The surface of the earth within the tropics becomes greatly heated by the sun, and this heat is communicated to the air in contact with the earth, causing it to ascend. To replace this air, a current of colder air sets in from the polar and temperate regions, thus tending to produce a north wind in the northern hemisphere, and a south wind in the southern hemisphere. To determine the actual direction of the wind it is, however, necessary to consider the rotation of the earth from west to east. A point on the surface of the earth near the equator has a velocity, from west to east, of more than a thousand miles per hour; but this velocity decreases as we approach the poles, where it becomes zero. Hence, if a mass of air starts from a place having a velocity, from west to east, of eight hundred miles per hour, and flows towards the equator, its direction relative to the surface of the earth will gradually tend *towards the west*, until, on reaching the equator, it will have a **relative** velocity of many miles per hour *from east to west.*

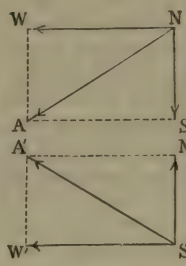


Fig. 95.

Hence, if, at any point on the borders of the temperate and tropical zones in the northern hemisphere, N S (Fig. 95) represents the velocity of the current of air towards the equator, and N W represents its relative velocity towards the west, then N A represents in magnitude and direction the actual velocity of the wind relative to the surface of the earth. Similarly S′ A′ represents the magnitude and direction of the wind in the southern hemisphere. Thus, a north-east wind* is produced in the northern hemisphere, and a

* A north-east wind is a wind *from* the north-east.

south-east wind in the southern hemisphere. These winds are known as the **trade winds**.

The directions of the trade winds of course vary with the latitude and the shape of the surrounding coasts. Students should consult their atlases to see how far theory agrees with fact.

Convection currents in the ocean are but feeble. Nearly all the ocean surface currents, the Gulf Stream included, are due to the winds blowing along the surface.

155. Heating by hot-water pipes. Another application of convection currents is found in the system of heating buildings by hot-water pipes. The principle is the same as that of Exp. **108**. The water in the boiler, which is placed at the lowest available position, is heated directly by the fire and rises through the outflow pipe, which should emerge from the boiler at its highest point, and, circulating round the network of pipes, return by the return pipe which enters the boiler at its lowest point. The rapidity of the circulation depends on the difference between the average densities of the water in the outflow pipe, from the boiler to the highest point reached, and that in the return pipes from the highest point to the boiler. This difference of density itself depends on the difference of temperature, which should therefore be as great as possible, and for this reason the outflow pipe should pass vertically upwards as far as possible, so as to have a long vertical column of water at the highest possible temperature.

The transmission of heat by this system of hot-water pipes from the boiler furnace to a person feeling the warmth is a good example of the different methods of transmission of heat. The heat passes from the furnace to the water in the boiler by *conduction* through the boiler plates. It is transmitted through the mass of the water by *convection*, passes through the pipes to the air in the room by *conduction*, and this air, heated by *convection*, in its turn warms any person whom it surrounds. Heat is also *radiated* from the pipes to the different persons and objects in the room.

EXERCISES XV.

1. Wherein does the manner in which heat is usually diffused through liquids and gases differ from its mode of diffusion through solids?

2. The bulb of a delicate thermometer is immersed at a slight depth below the surface of water. The upper surface of the water is heated, and the thermometer is hardly affected. On heating the water from below, the thermometer shows an immediate rise of temperature. Explain these two results.

3. Two test tubes A and B are filled with water. A small piece of ice is allowed to swim in A, and a similar piece of ice is sunk by a weight to the bottom of B. Heat is applied to the closed end of A and to the open end of B. In which test tube may we expect the ice first to melt? and in which may we expect the water first to boil? Give reasons for your answer.

4. Explain the heating of a building by hot water circulating from a single boiler in the basement.

5. State the reasons of the following:—
   (*a*) The warmth of the sea at its surface compared with its coldness in its deeper parts.
   (*b*) The warmth of the atmosphere near the earth compared with its coldness at great elevations.

6. (*a*) Why do the lower parts of the ocean never get warm?
   (*b*) Why is the surface-water of the ocean sometimes salter than that below?
   (*c*) Why is ocean-water near land sometimes less salt than that far away?
   (*d*) Why are the warm currents, like the Gulf Stream, confined to the surface of the ocean?

CHAPTER XVI.

WORK AND ENERGY.

156. It is well at this stage that the student should become familiar with certain mechanical principles and theorems dealing with work and energy. Before proceeding to these it is perhaps best to recapitulate some elementary definitions. It is assumed that the student is familiar with the English and Metric systems of units and their relations to each other.

157. Matter. All bodies which are known to us consist of substances or materials of varying character. These substances or materials are included under the common name of Matter.

Mass. The quantity of matter in a body is called its mass.

Velocity. The velocity of a body in a certain direction is its rate of change of position along a line in that direction.

It follows from this that the space described by a moving body in a given time is equal to the product of the average velocity and the time.

Momentum. The momentum of a moving body is a property it possesses **by virtue of its mass and velocity** conjointly, and is measured by the product of its mass and its velocity.

Acceleration. The acceleration of a moving body is the rate of change of its velocity.

Thus the velocity of a falling body increases by 32 feet per second every second, *i.e.* starting from rest the velocities at the end of the 1st, 2nd, 3rd secs. are 32, 64, 96 ft. per sec. In the metric system this increase is 981 centimetres per second per second.

Force. Force is that which changes, or tends to change, a body's state of rest or motion.

This is the usual definition of force ; but the idea of force is really fundamental, and cannot be satisfactorily defined.

If a constant force acts on a body, it gives it an accelerating motion, and it is to get a numerical value of force that we define it as equal to the product of the mass and the acceleration. From this it follows that when a force is accelerating the motion of a body the force is numerically equal to the rate of change of the momentum of the body.

Weight. Since the earth pulls a body towards it with a constant acceleration the earth must exert a force upon that body. This force is called the weight of the body.

Carefully distinguish between mass and weight, care being taken to express the force and mass in the correct units.

This enables us to define the unit of Force.

The **F.P.S. unit of force** is that force which, acting on a mass of one pound, gives it an acceleration of one foot per second per second. It is called the **poundal.**

The **C.G.S. unit of force** is that force which, acting on a mass of one gramme, gives it an acceleration of one cm. per sec. per sec. It is called the **dyne.**

These are called the **Absolute Units of Force.**

It follows from the above that if m represent the mass of a body, w its weight, and g the acceleration due to gravity

$$w = mg.$$

In the English system of units, m is in pounds, g in feet per second per second, and w in pounds-weight.

In the Metric system of units, m is in grammes, g in centimetres per second per second, and w in grammes-weight.

Gravitational Units of Force. For every-day purposes the weight of a pound and the weight of a gramme are taken as the units of force in the English and Metric systems respectively. The value of these units at any one place depend upon the value of g, which slightly varies from place to place.

It follows from the above that

(1) A pound-weight is equal to 32·2 poundals, so that a poundal is very nearly equal to the weight of half an ounce.

(2) A gramme-weight is equal to 981 dynes, so that a dyne is very nearly equal to the weight of a milligramme.

158. The words "work" and "energy" are used in Physics with certain technical meanings which are explained in this chapter. The student must distinguish very carefully between the scientific use of these words and their use in ordinary language.

Work.

159. Work. Definition.—**Whenever a force acts upon a body in such a way that motion takes place, work is said to be done by the force.**

Examples. A horse drawing a cart along a rough road; a bricklayer carrying bricks up a ladder; a man drawing water at a well or pump—in all these cases work is done.

But, unless motion takes place, no work is done.

When a man lifts a stone up from the ground, he does work; if he further *holds* it up at a certain distance from the ground he *exerts force*, but he does not do any work.

Thus, **work** *is done* **by a force** *when its point of application moves in the direction of the force.*

On the other hand, *when a body upon which a force acts moves* (owing to other causes) *in a direction opposite to that of the force,* **work** *is said to be done* **against the force.**

Examples. When a body falls to the ground the force of gravity does work; when coal is hauled up a pit work is done against the force of gravity. An engine in drawing a train on the level does work against the friction of the rails and axles, the pressure of the wind, etc.; if the train moves up hill, work is also done by the engine against the weight of the train.

160. Measure of work. *The work done by or against a force is measured by the* **product** *of the* **force** (*in units of force*) *and the* **distance** (*in units of distance*) *through which its point of application has moved* **parallel to the line of action of the force.**

If, therefore, the force is F, and the body moves through a distance s parallel to the line of action of the force, the

$$\text{work done} = Fs.$$

The phrase "the distance parallel to the line of action of the force" requires further explanation. If a man hauls a piece of marble up to the top of a house 90 ft. high, he does work against the weight of the marble through a distance of 90 ft., for this 90 ft. is measured vertically, *i.e.* in the line along which the weight of the marble acts. If, however, he pulls the marble up an inclined plane of length 90 ft., s is not to be taken here as 90 when we are thinking of the work done against the weight. The weight acts *vertically*, and we must therefore inquire what is the *vertical* distance through which the marble has been raised. If the top of the plane is 55 ft. above the bottom, then work has been done against the weight of the marble through a distance of 55 ft.

161. Units of work. The **unit of work** is the work done by the unit force when its point of application is moved through unit distance parallel to its line of action. There are two units in each system, the **absolute unit** used by scientists and the **gravitational unit** used by practical engineers, who find the absolute unit too small to be convenient.

162. Absolute units of work. The F.P.S. unit of work is the work done by a poundal acting through a distance of 1 ft. This is termed a **foot-poundal.**

The C.G.S. unit of work is the work done by a dyne acting through a distance of 1 cm. This is termed an **erg.**

Examples. (1) If a force of 5 poundals moves a body through 4 ft. along its line of action the work done = Fs = (5 × 4) or 20 ft.-poundals.

(2) A force of 9 dynes acting through a distance of 2 metres does (8 × 2000) ergs, since 2 metres = 200 cms.

163. Gravitational or practical units of work. These are (1) The **foot-pound (ft.-lb.)**, which is the **work** *done by a force of* **1 lb. wt.** *acting through a distance of* **1 ft.**, *i.e.* the **work** *done in raising a mass of* **1 lb.** *through a vertical distance of* **1 ft.**

The foot-pound is equal to 32 foot-poundals.

(2) The **kilogrammetre**, which is the **work** *done by a force of* **1 kilogramme wt.** *acting through a distance of* **1 metre**, *i.e.* the **work** *done in raising a mass of* **1 kilogramme** *through a vertical distance of* **1 metre**.

The kilogrammetre is equal to 981000 ergs.

Example. What work is done in raising a hundred-weight of coal 6 feet?

The force overcome = the weight of 112 lbs.,

and the distance along the line of action of the force = 6 *ft.*

$\therefore$ work done = $Fs = (112 \times 6)$ *ft.-lbs.* = 672 ft.-lbs.

NOTE that the answer is in ft.-*lbs.*, because we expressed the forces in *lbs.* wt.

164. Power. The power of a body is the **rate** at which it can work. The theoretical unit of power is therefore that which can do a foot-poundal of work in the English system or an erg in the metric system per second.

The practical unit of work in English-speaking countries is the **Horse-Power.**

Watt assumed an average horse could do 33,000 ft.-lbs. of work per minute, and this has gradually become the unit in which the power of all engines, motors, waterfalls, etc., are expressed.

Example. If an engine is capable of doing 11,000 ft.-lbs. of work per second, its H.P. is $\frac{11000}{550}$, *i.e.* 20.

The practical unit of power in the metric system is called the **Watt.** It is the power of an agent which works at the rate of ten million (10^7) ergs per second. A larger practical unit is the **Kilowatt**, which is equal to a thousand watts. One H.P. = 746 watts, $\therefore$ a kilowatt = about $1\frac{1}{3}$ H.P.

ENERGY.

165. Energy. DEFINITION.—**The energy of a body is the quantity of work it is capable of doing.**

Energy refers to the *total quantity* of work the body can do, and implies nothing as to the time in which the work is done.

166. Kinetic energy. DEFINITION.—*The* **kinetic energy (K.E.)** *of a body is* **the quantity of work** *it is capable of doing* **by virtue of its mass and its velocity** *conjointly.*

A cyclist gets up speed before he comes to a hill, and his increased speed assists him in mounting the hill, and by means of it he does work against gravity.

Water in motion can turn a water-wheel and thus grind corn.

The **kinetic energy** of a body of mass m moving with mass m is given by half the product of its mass and the square of its velocity, *i.e.*

$$\text{K.E.} = \tfrac{1}{2}mv^2.$$

Proof. Let a force act on a body of mass m through a distance s. The velocity initially is zero. It increases uniformly to a value v, which is the final value.

The work done by the force is equal to the kinetic energy of the body (see Arts. 169, 176), but the work done by the force

$$\begin{aligned}
&= \text{force} \times \text{distance acted through} \\
&= (\text{mass} \times \text{acceleration}) \times (\text{average velocity} \times \text{time}) \\
&= (\text{mass} \times \text{average increase in velocity per sec.}) \times \left(\frac{\text{initial velocity} + \text{final velocity}}{2} \times \text{time}\right) \\
&= m \times \frac{v}{t} \times \left(\frac{o+v}{2}\right) \times t \\
&= \tfrac{1}{2}\, m\, v^2,
\end{aligned}$$

$\therefore$ the final kinetic energy of the body is equal to $\frac{1}{2}\, m\, v^2$.

If m is given in pounds and v in feet per second the K.E. is in foot-poundals.

To express the energy in foot-pounds divide by g, *i.e.* 32.

If m is in grammes and v in cms. per sec. then the K.E. is in ergs.

To express the energy in gm.-cms. divide by g, *i.e.* 981.

167. Potential energy. If a man throws a stone up to the top of a cliff, he does work against the weight of the stone. If the stone then lodges on the top of the cliff, it is in such a position that, if allowed to do so, *e.g.* by being pushed off the ledge, it will acquire a velocity, and therefore kinetic energy. By means of this kinetic energy it will be

able to do the same amount of work as the man did in throwing the stone up.

We see then that the stone is capable of doing work merely by virtue of its position relative to the earth, and to the work it can thus do is given the name *potential energy*.

DEFINITION.—**The potential energy (P.E.) of a body is the quantity of work it is capable of doing by virtue of its position.***

In the case of gravitational potential energy, this position is reckoned relative to the earth's, so that when the body is on that surface its potential energy is zero.

The **potential energy** of a body of mass m at a distance h about the ground is equal to the product of the weight of the body and the height, *i.e.* P.E. $= mgh$. The force required to lift it is equal to the weight of the body, and this force is exerted through a distance h.

The work done is, by Art. **160**, equal to mgh.

Assuming the potential energy when on the ground to be zero, mgh is therefore the potential energy of the body at the height h.

It follows from our definitions that the potential energy of a body at a given height is equal to the kinetic energy it would have just before reaching the ground if dropped from this height.

Exp. 111. Transformation of potential energy into kinetic energy. Hang a small weight by an elastic string from a support. Notice the position of the weight. Now raise the weight and let it fall. Notice that the weight descends below its first position, comes momentarily to rest at a definite spot, and then reascends, oscillating about its original position. The body, therefore, in stretching the string, does work against the tension of the string.

Deduction. In raising the weight, we gave it *potential energy*. When it reached the position in which at first it rested it was in motion and, therefore, had *kinetic energy*. Thus the *potential energy* had been transformed into *kinetic*. When it reached its lowest position it had lost its *kinetic energy*, and also some more *potential energy*, but the string now possesses *potential energy* due to *strain*.

---

* Potential energy may also be due to a change in the relative positions of the particles of a body, as in the case of the main spring of a clock.

Further illustrations. (i) A pile-driver before being allowed to fall has potential energy, which has been stored up in it by the men who pulled it up into position, and this energy is converted into kinetic energy when the pile-driver is released.

(ii) A lake up in the hills has potential energy. No work can be done by it, however, unless the water is allowed to run down, and thus acquire kinetic energy.

(iii) When a watch spring is wound up work is done upon it against the elasticity of the spring. When permitted to do so the spring will emit this work again in turning the wheels of the watch as it tends to unwind itself. This also is a case of potential energy. In this case, however, energy is imparted to the body (*i.e.* the spring), not by raising it but by distorting it.

(iv) Suppose two particles tied to a piece of elastic string and placed on a table so that the string is taut but not stretched. If the string be stretched and then set free, the particles move towards each other, *i.e.* the string employs force on the particles. Thus, in the stretched position, there was stored up in the string potential energy which was changed to kinetic energy of the particles when the string was released.

We see then that a body may possess potential energy (1) when it is raised without distortion above the level of the earth, (2) when its shape is distorted.

168. Energy can be transferred from one body to another. If a billiard ball in motion strikes one at rest, the latter begins to move whilst the former moves on with speed diminished. Thus the ball which had no energy before the impact possesses energy after the impact, whilst the ball originally in motion loses part of its energy. It follows that energy has been transferred from one ball to the other.

Impact is one method of transferring energy. Probably a more common way of bringing about such a transfer is illustrated as follows. Hold in the hand one end of a strong string to which a weight is attached. Move the hand to and fro in a horizontal line. No particle of the string moves, except to vibrate in a horizontal line, but the movement passes along the string until the particles near the weight and finally the weight itself are set in motion. Thus, without the transfer of any *material* from the hand to the weight, motion, and therefore kinetic energy, has been transferred by vibrating an intermediate medium. If

a bundle of strings were all tied to the weight and held by the hand, and if the hand were made to vibrate in the same way, exactly the same phenomenon would be witnessed. Thus, whether the intervening medium be as fine as a fine string or have a considerable section like the bundle of strings, energy may be transferred from one body to another without the transfer of matter. It is probably by means of the vibrations of some such intervening medium called the *ether* that the energy of the sun is transferred to us and makes itself sensible as heat and light.

169. Several forms of energy. There is reason to believe that the particles of every body, whether solid, liquid, or gas, are in perpetual vibration. Indeed the heat which bodies possess and emit is most probably due to the vibration of their particles. Owing to their vibration these particles possess kinetic energy and it is assumed that what is called heat is due to this energy.

It is now firmly held that not only heat, but also light and sound are only forms of energy. Great experimenters have shown (1) that any one form of energy can be converted into any other, (2) that in such a conversion the quantity used up of the one is invariably equal to the quantity of the other created. It is not difficult to show the transformation from one form to another by simple experiment, but to show the strict equality between the quantities used and created requires special apparatus. The subjects of the transformation of work into heat and *vice-versa* will be treated in detail in the next chapter.

EXERCISES XVI.

1. Define work. Give an illustration to show how a moving body may do work.

2. How much work is done in each of the following cases? Give the answers in appropriate units.

(*a*) When the mass of 10 lbs. falls 6 ft.

(*b*) By a man of 10 stone weight walking upstairs to a height of 50 feet.

(*c*) By a force of 4 poundals acting through a distance of 10 yards.
(*d*) By a machine pulling a million grammes of coal up a pit a thousand metres deep.
(*e*) By a force of 3 dynes acting through 2 centimetres.

3. When is the energy of a pendulum bob (i) wholly potential, (ii) wholly kinetic?

4. When an arrow is on the point of release the bow possesses potential energy. What is the difference between this case of potential energy and that of a body raised above the level of the earth?

5. A bullet leaves a gun with a velocity of 800 feet per second The length of the barrel is 30 inches and the mass of the bullet is 1 oz. Find (1) the kinetic energy of the bullet when it leaves the muzzle; (2) the average force on the bullet while in the barrel due to the powder, assuming that the force is uniform.
Express the force in poundals, pound-wts., and in dynes.

CHAPTER XVII.

THE MECHANICAL EQUIVALENT OF HEAT.

170. Introductory. It has already been stated that heat is a form of energy, and that a given quantity of heat, measured in any arbitrary units, has a definite and constant equivalent in mechanical work or energy measured in mechanical units such as foot-pounds or ergs. We shall now briefly sketch the experiments and arguments which establish the truth of this dynamical theory of heat.

Up to the commencement of the present century heat was considered to be a material "igneous fluid" called *caloric*, which, according as it was present in a substance in large or small quantities, rendered that substance hot or cold. When it was shown that heat could be produced by friction, percussion, and other mechanical means, this was explained by assuming that these mechanical actions changed the condition of some portion of the body, and thus diminished its capacity for heat; or, in plainer terms, some of the caloric was forced out of the body and made to appear as sensible heat.

The dynamical theory of heat was established by two distinct steps. Towards the close of the last century the experiments of Rumford and Davy showed that *heat could not be a material substance*, and thus the old caloric theory of heat was disproved. This theory, however, still held the field until in 1849 the results of Joule's quantitative experiments established *the equivalence of heat and work, and proved that heat must be a form of energy*. Later experiments by other experimenters and the mathematical development of the subject by Rankine, Thomson, and Clausius entirely confirm this result, and completely establish the dynamical theory.

171. Rumford's experiments. Count Rumford was an Anglo-American in the employ of the Bavarian Government, and was led to experiment on this subject whilst superintending the boring of cannon at the Munich Arsenal. He noticed that during boring a large amount of heat was developed both in the cannon and in the boring-bar, and, struck with the apparently unlimited supply of heat, he determined to try and find out whether heat was really a material fluid (*caloric*), as it was then considered to be.

In considering the method of his experiment, it occurred to Rumford that if the heat was forced out of any part of the system he had to deal with it came most probably from the borings, and accordingly he decided to determine whether there was any difference between the borings and the solid metal in capacity for heat. This he did by taking equal weights of each, heating them to 212° F., and then immersing them separately in equal quantities of water at the same temperature. After one minute the rise of temperature in each case was the same. This experiment he repeated several times with the same result, and finally came to the conclusion that the heat produced could not have been furnished at the expense of the borings. Still pursuing the same train of thought, Rumford determined to use a *blunt* borer, so that only a very small quantity of borings should be produced in a given time. He arranged apparatus for this experiment, and found that an enormous quantity of heat was still evolved, although the weight of the borings produced was only about one thousandth of that of the whole apparatus.

Rumford was now convinced that the heat could not come from the borings; neither could it come from any other part of the apparatus, for the metal was still in its initial condition, and must therefore have the same capacity for heat, and every part of the system was *gaining* heat instead of losing it.

It then occurred to him that the only other source from which the heat could come was from the air. He accordingly excluded the air by causing the boring to take place under water. In two and a half hours from the commence-

ment of the experiment the water boiled. This heat, it was argued, could not have come from the water, for the only change of condition experienced by the water was that due to *gain* of heat. From these experiments Rumford came to the conclusion that heat was *not* a material substance. In a paper published in 1798 he states his case thus: "Anything which any insulated body, or system of bodies, can continue to furnish *without limitation* cannot possibly be *a material substance*; and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in these experiments except it be *motion*."

172. Davy's experiments. In 1798 Sir Humphry Davy showed that when two pieces of ice are rubbed together in a vacuum the ice melts, even if the initial temperature of the ice and its surroundings be some degrees below the freezing point. The melting of the ice shows that heat is produced, and its production cannot be due to a diminution of thermal capacity, for the capacity of the water produced is *greater* than that of the ice. Davy did not at the time see the full significance of this result, and it was not until 1812 that he clearly stated his opinion that heat was a form of molecular energy.

Exp. 112. Hammer a piece of lead, saw wood, etc., and test the temperature of the lead, saw, etc., before and after the experiment. It will be found that the temperature has risen.

Exp. 113. Rub a brass nail or button on a wooden seat, and notice its increase in temperature.

Exp. 114. Bore a hole in a piece of hard wood with a brad-awl. The brad-awl becomes hot.

Exp. 115. Feel the axle of a wheel which has been rotating rapidly. It is warm, possibly hot.

Exp. 116. Take a piece of thin iron plate and bend it forwards and backwards to produce fracture along a certain line. Feel the metal at this line. It is hot.

Deduction.—In these experiments the mechanical work expended in impact or friction has been transformed into heat.

Exp. 117. Use a bicycle pump to inflate the tire or pump air into a closed bottle. Note that the pump becomes warm. Part of this is due to friction between the piston and the barrel; but the greater part is due to the heat developed in the air on compression.* This heat is equivalent to the work done in compressing the air.

Study the action of an engine. Note the transformation of the energy of the heat of the burning coals into the energy of mechanical motion.

173. Joule's experiments. Notwithstanding the conclusiveness of Rumford's and Davy's experiments, they excited but little interest at the time, and the material or caloric theory continued to be held by most physicists up to the year 1840. It was not, however, till 1849 that the dynamical theory was generally accepted. In this year Dr. Joule, of Manchester, completed a series of experiments, which not only showed that heat was a form of energy, but accurately determined the quantitative relation between a given quantity of mechanical energy and the equivalent quantity of heat.

Joule's most important experiments were on the development of heat by fluid friction and on the heat effects associated with the compression and expansion of gases. Some of these experiments are described below.

174. Joule's experiments on fluid friction. The most important of Joule's experiments consisted in the development of heat by churning water in a copper vessel by means of a suitable paddle. The friction of the water on itself and on the surfaces of the vessel and paddle produced heat, which resulted in an appreciable rise in the temperature of the water.

The apparatus used in this experiment is shown in Fig. 96. C represents the vessel containing the water. It was made of copper, and was capable of containing about a gallon and a half. Inside it were fixed vertically, and at right angles to each other, four vanes or partitions,

* To test this point work the pump with the nozzle open, the pump becomes only slightly warm.

arranged as shown in the section given in Fig. 97. The vanes were attached to the brass framework which supported the sockets for the axle of the paddle, in such a

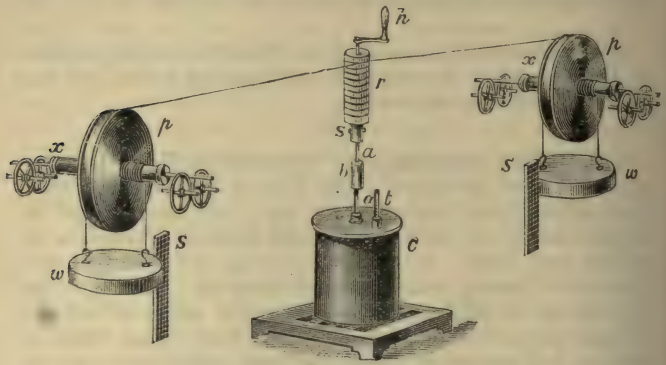


Fig. 96.

way as to leave a free central space in which this axle revolved. The paddle consisted of a central axle carrying eight sets of brass vanes similar to those shown in Fig. 97.

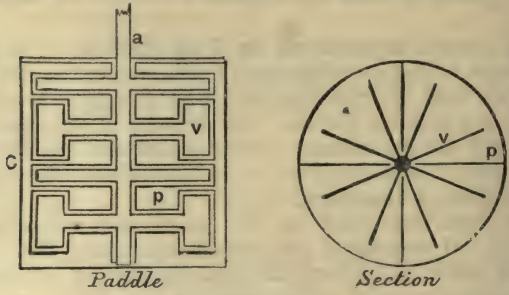


Fig. 97.

The axle, *a a*, revolved in two sockets at the top and bottom of the vessel C, and the revolving vanes passed through spaces cut in the stationary vanes, just as a key

turns in the wards of a lock. By this arrangement the water was prevented from rotating in the direction of revolution of the paddle.

The motion of the paddle was produced by the fall of two equal leaden "weights," *w w*, attached by strings to the equal axles, *x x*, of two exactly similar and equal wooden pulleys, *p p*. The motion thus communicated to the axles (2 inches in diameter) was transmitted by the pulleys (1 foot in diameter), by means of cords, to the vertical wooden spindle, *r*, attached to the upper part of the paddle axle. To diminish friction as far as possible, the pivots of the pulleys were of steel, and rested each on two steel friction wheels. The handle, *h*, was used to wind up the weights into the position shown in the figure; by taking out the pin at *s* the spindle *r* could be disconnected from the axle, *a a*, and the weights could thus be wound up without causing rotation of the paddle. With this apparatus Joule set himself to determine the quantity of heat produced by the expenditure of a given amount of energy. For this purpose the water equivalent of C and its accessories was determined, and a known quantity of water was placed in C. The temperature of this water was given by a very delicate thermometer, *t*, reading to $\frac{1}{200}$th of 1° F. To prevent loss of heat from C it was placed on a wooden grating which touched the ground only at the four corners, and the axle, *a a*, which was of brass, was made in two pieces, joined at *b* by a piece of wood. These precautions helped to prevent loss of heat by *conduction*.

The weights were now wound up, the pin at *s* placed in position; and the temperature indicated by *t* having been read, the weights were allowed to fall to the ground. This was repeated twenty times with as little delay as possible, and the final temperature of the water in C was then carefully taken. It was noticed that the velocity of the weights during the latter half of their fall was nearly uniform. This velocity was determined by repeated observation with some accuracy. The data now obtained, subject to a few corrections, served to determine the required relation.

Let m denote the mass of each weight, h the height of its fall, and v the velocity on reaching the ground; then

$$w = m\,g\,h - \tfrac{1}{2}\,m\,v^2$$

denotes in absolute units* the work done by each weight during each fall, for $m\,g\,h$ is the loss of potential energy of each weight and $\frac{1}{2}\,m\,v^2$ is the gain in kinetic energy of each weight. Hence if the weight fall n times, the total work done is given by $2\,n\,w$. Again, let θ denote the rise of temperature indicated by the thermometer t, and M the water equivalent of the apparatus C and the contained water; then

$$\mathrm{Q} = \mathrm{M}\,\theta$$

denotes the heat produced by the expenditure of a quantity of work denoted by $2\,n\,w$.

The quantity $2\,n\,w$ is subject to several corrections, the most important of which is that due to the work absorbed by friction outside the water. To determine this correction, Joule disconnected the spindle from the paddle, and connected the weights by a single cord adjusted round r in such a way that, as one weight descended, the other was raised. A small weight, p, was then placed on one of the weights, and adjusted by repeated experiment until the velocity of descent was equal to that noted during an actual experiment. The weight p thus approximately represented the resistance due to friction and the work spent in overcoming it, for each descent is given by $w' = p\,h$. Hence, the work absorbed by friction during n descents is given by $n\,w'$, and the total work expended in the production of heat in the water is expressed by $\mathrm{W} = 2\,n\,w - n\,w'$. The quantity Q is also subject to an important correction due to the loss of heat by radiation during the thirty-five minutes occupied by the experiment. This was corrected by noting the rate of cooling for thirty-five minutes after the completion of the experiment. Let q denote the loss of heat by

* In gravitational units the equation reads $w = \mathrm{M}\,h - \frac{1}{2}\frac{\mathrm{M}}{g}v^2$, where M is the weight of each weight.

radiation; then $(Q + q)$, which may be denoted by H, expresses the total amount of heat produced by W units of work.

The object of Joule's experiments was to determine the relation between these two quantities. If H thermal units of heat are equivalent to W units of work, then one thermal unit of heat is obviously equivalent to $\frac{W}{H}$ units of work.

Joule determined the value of this ratio from a very large number of experiments. He found it practically *constant*, thus establishing the principle of equivalence between work and heat, and he gave as the mean value of the ratio—

$$\frac{W}{H} = 772,$$

where W is expressed in foot-pounds and H in pound-degrees Fahrenheit. This means that one pound-degree Fahrenheit of heat is equivalent to 772 foot-pounds of work, or that if 772 foot-pounds of work be expended in the production of heat the quantity of heat produced would raise one pound of water through one degree Fahrenheit.

The value of this ratio, $\frac{W}{H}$, is known as *Joule's equivalent* or the *mechanical equivalent of heat* and is usually denoted by the letter J. It might also be called the *specific heat of water in mechanical units.*

Joule also carried out a series of experiments on the friction of mercury, using a small *iron* apparatus similar to that described above. The results obtained were practically identical with those obtained with water, thus proving that *the equivalence of heat and work is independent of the nature of the apparatus by which the transformation is effected.* This conclusion was further confirmed by the results of some experiments with iron plates producing heat by their friction when rubbed together under mercury, and also by experiments on the compression and expansion of gases.

Much work on the same subject has been done since the time of Joule, with the result that Joule's figures have been shown to be slightly inaccurate. The latest figures for J are given in the following table:—

778 foot-pounds per water-pound-degree Fahrenheit.

1400 foot-pounds per water-pound-degree Centigrade.

·427 kilogram-metre or 427 gram-metres per water-gram-degree Centigrade.

$4{\cdot}19 \times 10^7$ ergs per water-gram-degree Centigrade.

10^7 ergs is often called a **joule**. Hence we may also write—

$$7 = 4{\cdot}19 \text{ joules per water-gram-degree C.}$$
$$\text{or } 4{\cdot}19 \text{ joules per calorie.}$$

Exp. 118. Make a rough determination of the value of the mechanical equivalent of heat. Obtain a *papier mâché* tube about 2 metres long and 5 cm. in diameter, and two corks which tightly fit the tube. Take a pound of small lead shot, pour them into a basin, and insert a thermometer in their midst. Wait ten minutes for the lead to acquire the temperature of the room. Take this temperature. Then pour the lead shot into the tube, close the tube and hold it vertical. Quickly invert it so that the shot fall the whole length of the tube. Repeat this operation 100 times, making sure that the shot fall the whole length of the tube each time. Then remove one cork from the tube and pour the shot into the basin and take their temperature as soon as possible.

From the rise of temperature calculate the heat developed, also calculate the work done.

Let m = mass in gm. of the shot,
h = average distance in cm. the shot fall each time,
n = number of times the tube is inverted,
s = sp. ht. of lead,
t = rise of temperature in degrees C.

Then work done by gravity $= n\,(m \times h)$ gm.-cm.,
heat gained by lead $= m\,s\,t$ calories.

$$\therefore \quad \mathrm{J} = \frac{n\,m\,h}{m\,s\,t} = \frac{n\,h}{s\,t} \text{ gm.-cm. per calorie}$$
$$= \frac{n\,h}{s\,t} \times 981 \text{ ergs per calorie.}$$

In the ordinary laboratory method of finding J the apparatus of Puluj is used. It comprises two steel cones,

the outer one screwed to a revolving shaft, while the inner one, filled with mercury or water, is free to move according to the amount of friction between the two. The mercury or water becomes heated by friction between the two cones, and thus the ratio $\frac{w}{h}$ can be determined. It would be tedious to give here full particulars for carrying out the experiment, and most instrument makers supply full details with the apparatus.*

175. The first law of thermo-dynamics. This law asserts the equivalence of heat and work in a definite ratio determined by experiment. It may be stated thus:—

When work is transformed into heat, the quantity of heat produced is equivalent to the quantity of work expended in its production; and conversely *when heat is transformed into work, the quantity of work produced is equivalent to the quantity of heat expended in its production.* That is, in both cases,

$$W = J H,$$

where W denotes any definite quantity of work, H the equivalent quantity of heat, and J the ratio of these two quantities. This ratio is known as **Joule's equivalent,** or **the mechanical equivalent of heat,** or **the specific heat of water in mechanical units.**

176. Conservation of energy. As we have already mentioned in Art. **169**, it has been proved that whenever energy in one form disappears it is always replaced by an exactly equivalent amount of other forms of energy. Hence, when all the forms of energy are taken into consideration—

(1) *The total amount of energy in the universe is absolutely unalterable,* i.e. *no energy is ever created or destroyed.*

* See special pamphlet issued by W. G. Pye and Co., Granta Works, Cambridge.

(2) *The various forms of energy may, though not always at will, be converted one into another.**

Energy may be transferred from one body to another, or transmuted from one kind to another, but the energy lost by one body is gained by other bodies, and *vice versa*.

The principle contained in the above propositions is known as the **Conservation of energy.**

177. Work done when a gas expands. Consider a piston in a cylinder separating two quantities of gas at different pressures. The result will be that the piston will be driven by the high-pressure gas towards the low-pressure gas. We want now to find how the work done by the gas depends on the pressures of the two quantities of gas—the area, A, of the face of the piston and the distance, l, the piston moves.

Let the gas pressures be $P + p$ and p respectively, the higher pressure being on the left of the piston (Fig. 98), and suppose that the pressures remain constant as the piston moves. Then the force exerted by the high-pressure gas on the left face of the piston is $(P + p)$ A,† and the force exerted by the low-pressure gas on the right face of the piston is p A. The resultant force is therefore P A directed from left to right. The work done by this force in moving the piston a distance $l = P A . l = P . A l$. Now A l is the volume swept out by the piston, therefore the work done = P × Volume swept out by the piston.

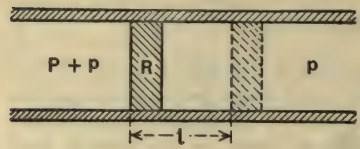


Fig. 98.

* In nearly all operations some energy is wasted in friction, etc., going into the form of heat energy; as a rule, very little of this energy is recoverable by human agencies, and it is therefore said to be *dissipated*. In any store of energy that portion which can be used for actual mechanical work is often spoken of as the *available energy*.

† Pressure = force per unit area; hence force = pressure × area.

If the gas to the right of the piston is air open to the atmosphere, p is equal to the atmospheric pressure, and P is therefore the pressure of the gas to the left of the piston in excess of the atmospheric pressure. This excess pressure is often called *the* pressure of the gas, and the above deduction may thus be expressed: "If a piston is moved along a cylinder by the action of a gas at constant pressure the work done by the gas is measured by the product of the pressure of the gas into the volume swept out by the piston."

178. Joule's experiments on the Compression and Expansion of gases. When a gas is compressed, the work done on it in effecting the compression produces heat, which raises the temperature of the gas. Similarly, when a gas expands, it does work against the external pressure, and consequently there is an absorption of heat, which lowers the temperature of the gas. (**Exp. 117.**)

Without experiment it cannot be assumed that *all* the work spent in compressing a gas is converted into heat, or that the heat absorbed during the expansion of a gas is equivalent to the work done. For example, when a steel spring is compressed, nearly all the work done in effecting the compression is spent in giving the spring potential energy, only a small fraction being spent in heating the steel. Similarly, when a gas is compressed the work done in compression may be partly spent in imparting potential energy to the gas, and partly in the production of heat. This point was put to experiment by Joule, who found that practically all the work done in the compression of a gas was spent in the production of heat in the gas, and that when a gas expanded nearly all the work done by the gas was at the expense of its own heat.

To find the mechanical equivalent of heat, Joule adopted the method of allowing a gas at high pressure to expand. In one of his experiments he employed a reservoir containing air under a pressure of 22 atmospheres, and immersed in water contained in a large calorimeter. The air escaped from the reservoir through a stopcock constructed to regulate the velocity of its flow, and through a long coil of lead

tubing, to the exterior, where it was collected in a large vessel under the pressure of the atmosphere. The stopcock and tubing were both immersed in the calorimeter, so that the air arrived at the exterior at the temperature of the calorimeter, and without any appreciable velocity.* The temperature of the water in the calorimeter was taken before and after the expansion; and the fall of temperature, multiplied by the water equivalent of the calorimetric apparatus, gave the heat, H, absorbed by the expansion of the air.

To determine the work, W, to which this is equivalent, it is necessary to calculate the external work done by the gas during its expansion. Now the gas expands against the pressure of the atmosphere, which is constant during an experiment of short duration; hence, by Art. 177, the work W done by the gas is equal to the product of the atmospheric pressure, P, and the volume, V, of the air collected after escaping from the reservoir and measured at atmospheric pressure.

That is $$W = PV,$$

and then $$W = JH,$$

from which J can be calculated.

From several experiments, of which the one described above is one of the simplest, Joule deduced results which agreed closely with those he had previously obtained by frictional methods.

179. Calculation of Joule's equivalent from the values of the two specific heats of gases. We have already, in Art. **74**, referred to the fact that the specific heat of gases at constant pressure is greater than that at constant volume. This is evidently due to the fact that when a gas is heated at constant pressure it expands, and part of the heat absorbed is expended in doing external work, *the remainder* being spent in increasing the temperature of the gas. When,

* Had the air been allowed to expand freely into the air, it would have carried away energy in the form of kinetic energy, and thus the fall of temperature would have indicated a greater absorption of heat than that equivalent to the external work done.

however, a gas is heated at constant volume, *no* external work is done, and *all* the heat absorbed is spent in increasing the temperature of the gas. From this it follows that the difference between the two specific heats should be equivalent to the external work done during the expansion, at constant pressure, of one gramme of the gas for one degree rise in temperature.

That is, if

s_p denote the specific heat at constant pressure,

s_v the specific heat at constant volume,

$w =$ the work done during expansion of 1 gm. of gas per 1° rise in temperature, and

h the heat equivalent of this work,

$$s_p - s_v = h = \frac{w}{\mathrm{J}}.$$

Hence, if s_p, s_v, and w are known J can be calculated.

The value of s_p may be found by the method of Art. **74**. To find s_v the steam calorimeter (Art. **121**) has been employed, the gas, at high pressure, being contained in a copper globe suspended from the beam of a balance. In the case of air $s_p = 0{\cdot}2410$ and $s_v = 0{\cdot}1715$, whence $h = {\cdot}0695$. To calculate w we require the increase of volume of 1 gm. of air per 1° C. rise of temperature and the pressure at which this occurs.

Take the pressure as 76 cm. of mercury at 0° C., in absolute units, this is $76 \times 13{\cdot}6 \times 981$, or $1{\cdot}014 \times 10^6$ dynes per sq. cm.

At 0° C. and a pressure of 76 cm. of mercury 1 litre of air weighs 1·293 gm. (Regnault), therefore the volume of 1 gm. of air at this temperature and pressure is $\frac{1000}{1{\cdot}293} = 773$ c.c., and the increase of volume for 1° C. rise of temperature $= \frac{773}{273} = 2{\cdot}83$ c.c.

Hence

$$w = 1{\cdot}014 \times 10^6 \times 2{\cdot}83 \text{ ergs.}$$

$$\therefore \mathrm{J} = \frac{1{\cdot}014 \times 2{\cdot}83}{{\cdot}0695} \times 10^6$$

$$= 4{\cdot}18 \times 10^7 \text{ ergs per calorie.}$$

180. Expenditure of the Latent Heat of Vaporisation on internal and external work. During the change of state from liquid to vapour there is a large increase of volume, and therefore when a liquid boils the vapour has to push back the atmosphere and so do work. The question arises, How much of the "latent" heat of vaporisation is given to increasing the molecular energy of the substance and how much is spent as work in forcing back the atmosphere? To answer this, the method of the preceding articles may be used; the following is an easy variation of this method.

Take the case of water. One gram of water at 100° C. occupies a volume of 1·043 c.c., and one volume of steam occupies at 100° C. and normal atmospheric pressure (76 cm. of mercury) a volume of 1672·5 c.c. (Art. **147**). Imagine, then, a long vertical cylinder of unit cross section provided with a weightless piston and suppose that the piston just encloses 1 gm. of water in the cylinder. Let us replace the atmospheric pressure by a weight placed on the piston. It will then be easier to see what work is done when the water turns into steam. The normal atmospheric pressure being that of 76 cm. of mercury the load of the piston should be $76 \times 13{\cdot}596$ or 1033 gm.

When the water at 100° C. turns into steam at 100° C. the piston is forced up a distance of $1672{\cdot}5 - 1{\cdot}04$ or approximately 1671 cm. The work done on the piston is

therefore 1033×1671 gm.-cm.

or $1033 \times 1671 \times 981$ ergs,

and the heat equivalent of this is $\dfrac{1033 \times 1671 \times 981}{4{\cdot}19 \times 10^7}$

calories, which works out to be 40 calories. Therefore of the 540 calories which are required to turn 1 gm. of water at 100° C. into one gram of steam at 100° C., 40 calories are spent on external work and the balance of 500 on increasing the energy of the molecules.

Other cases of vaporisation may be treated in the same way. The simple case of gas expansion on heating has been treated already in Art. **179**.

EXERCISES XVII.

1. When the tyre of a bicycle is being inflated, the pump becomes very hot. What is the source of this heat?

2. Why does the barrel of a gun become hot after several shots have been projected?

3. Describe how the energy of water which falls from a height into a pond without outlet is transformed.

4. Describe the transformations of energy when a spark is produced by means of a flint and steel.

5. Describe the transformations through which energy passes as clouds are formed and as rain returns from them to the earth.

6. Describe the passage of energy from the coal to the passengers and the rails when a train is in motion. Where else has energy gone?

7. In a ship-building yard a machine pierces holes in iron plates by punching out circular fragments. Before beginning work the machinery and iron plates are quite cold. After the operation the circular fragments are too hot to hold in the hand. Why is this? How could you ascertain experimentally the amount of heat gained by one of the fragments during the process?

8. What is meant by the mechanical equivalent of heat? In what sense can it be called the specific heat of water in mechanical units?

9. The specific heat of ice is 0·5. Express this (1) in foot-pounds, (2) in ergs.

10. A waterfall is 1,400 feet high. Assuming that no energy is lost by the water, find how much hotter the water at the bottom is than the water at the top. Express the answer on both the Centigrade and the Fahrenheit scale.

11. A 10-horse-power engine does 330,000 foot-pounds of work per minute. Assuming that only $\frac{1}{100}$th of the energy in the coal is utilised for mechanical purposes, and that one pound of coal generates 12,000,000 foot-pounds of work during combustion, find the mass of coal used per hour.

12. Express the latent heat of fusion of ice and the latent heat of vaporisation of water in foot-pounds and in ergs.

13. A pound of lead is lifted up 20 feet and dropped upon a stone slab. This is done quickly 10 times and at the end the lead is found to have warmed 4·65° C. above its initial temperature. Assuming that all the heat generated remains in the lead and that the specific heat of lead is 0·031 (water being taken as unity), calculate the work equivalent of the heat which would warm 1 pound of water 1 degree Centigrade.

14. When a gramme of water at 100° C. is turned into a gramme of steam at 100° C. and atmospheric pressure, the increase of volume is 1,671 c.cs. The pressure of the atmosphere being $1{\cdot}014 \times 10^6$ dynes per sq. cm., find roughly what fraction of the latent heat of vaporisation is spent in lifting up the atmosphere.

CHAPTER XVIII.

RADIATION.

181. Nature of radiation. As we have already mentioned in Art. **138**, radiation is different from conduction and convection. Heat is said to pass by radiation when it is transmitted from one body to another, through a vacuum, or through an intervening material medium without affecting the temperature of the medium. The heat of the sun reaches us by radiation, as does also the warmth we experience when we stand before a fire. A thermometer suspended in a vessel from which the air has been exhausted receives heat from external objects; which shows that the presence of air is not necessary to the passage of heat by radiation. Again, a piece of black cloth, if exposed to the rays of the sun on a frosty day, becomes much warmer than the surrounding air. This shows that heat is able to pass through a medium like air without warming it nearly so much as it does some object in the medium or beyond it.

Other gases besides air have been found to readily allow the passage of heat by radiation. The drier a gas is the more readily does heat traverse it by radiation. It was mentioned that heat travels through a vacuum, and experiments have shown that it does so better than through the driest air.

Many solids and liquids allow radiant heat to pass through without being themselves warmed to any great extent: *e.g.* rock salt and bisulphide of carbon. Other bodies, such as wood, slate, metal, etc., do not permit its passage; they absorb the energy, and therefore become heated.

Various experiments have proved that radiant heat is transmitted in the same way as light. In the first place, most bodies which are transparent to light are more or less transparent to "radiant heat." Again, the speed with which heat is radiated from hot bodies is the same as that with which light travels from luminous bodies. In a total eclipse of the sun, for example, both heat and light are cut off simultaneously. Also, heat is reflected from polished surfaces, and the laws of its reflection are the same as in the case of light. It is also refrangible, and can thus be brought to a focus by a lens. The use of a simple lens as a burning-glass shows that the heat which has passed through the lens is concentrated at about the same spot as the rays of light.

Since heat is radiated from one body to another through a vacuum, and as the radiation of heat bears so many analogies to the passage of light, it is believed that both are transmitted by means of the undulations of the ether. As we use the term *rays of light*, so we may speak of *rays of heat* in a similar sense.

The molecules of a white-hot body are in a state of rapid vibration, the frequencies of vibration being very different for molecules of different substances and for the same molecules at different temperatures. These moving molecules disturb the ether around them and send waves of disturbance through it. The velocity with which these waves travel through the ether is the same. All waves, whatever their frequency, warm bodies when they are absorbed. The most familiar hot bodies, including the sun, send out most energy in vibrations of lower frequency than visible light. If the body is black like lampblack it absorbs all the incident radiation, and hence the energy of all the waves is spent in heating the body. The several properties of the waves which depend more or less upon the frequency are, however, more fully described in text-books of Light.*

A better term for *radiant heat* is *radiant energy*, but the word radiant itself is not a very good word, for it implies the existence of radii or rays along which energy is transmitted, whereas we now believe energy is transmitted in spherical or circular waves, just as the waves of disturbance in a pond of water in which a stone is thrown travel out in ever-enlarging circles.

We shall, however, use the term radiant heat for waves of low

* See *The Senior Sound and Light*: Light, Art. 101.

frequency that do not excite vision. Another name is infra-red rays, as these rays are refracted by a prism to a less extent than the red rays.

When a body is first heated it sends out waves of low frequency (or long wave-length), the infra-red waves. As the heating is continued waves of higher frequency are added to these, and the waves added just when the temperature rises as far as 500° C. are just quick enough to excite in our eyes the sensation of red light. As the heating is continued still further waves which excite the sensations of orange, yellow, green, blue, and violet light are successively added to the other waves, so that the colour of the body gradually changes, the waves exciting the violet sensation being added at about 1300° C. The body now appears white-hot. Further heating does not alter the colour, but enables the body to send out waves of very high frequency which have intense chemical and electrical effects. These very high frequency waves are called actinic or ultra-violet waves. They are refracted by a prism more than the violet waves. All these waves carry energy, and have an equal claim to the title of Radiant Heat. But this name is sometimes given specially to the infra-red waves because these carry such a large proportion of the heat emitted by ordinary sources of light and heat; and the whole emitted as radiation by hot water, etc.

182. The behaviour of bodies towards radiant heat. When radiation is incident on the surface of any medium, it is, in general, split up into four parts, viz.:—

1. A portion which suffers **regular reflection** at the surface of the medium in accordance with the ordinary laws of reflection.

2. A portion which suffers *reflection* at minute irregular portions of the surface. This is called **diffusion**.

3. A portion which is *refracted* into the medium in accordance with the laws of refraction.

A part of this is

(*a*) **transmitted** by the medium; and the other portion is

(*b*) **absorbed** by the medium.

A body which transmits nearly all the radiant heat which is incident on it is said to be **diathermanous**. A body which absorbs nearly all the incident radiant heat is said to be **adiathermanous**. It follows from a consideration of the energy of the radiation that the passage of radiant heat through a body does not raise its temperature

if the body is perfectly diathermanous, but does raise it if the body is adiathermanous.

If the total quantity of incident radiation is represented by unity, and the quantities reflected, diffused, transmitted, and absorbed by r, d, t, and a respectively, then we have

$$r + d + t + a = 1.$$

The quantities in the left-hand expression are essentially positive; and it is therefore evident, from the relation given, that if one of these quantities is large—nearly equal to unity—then the others must be small. For example, if $r = 0{\cdot}9$, then d, t, and a must be each less than $0{\cdot}1$. That is, if the reflecting power of the surface of a medium is very great, then its powers of diffusion, transmission, and absorption are small. Further, it is evident that any one, or the sum of any two or three, of these quantities may be equal to unity, and thus the remaining quantities or quantity must vanish. That is, the incident radiation may, for example, be entirely absorbed by a medium, or completely reflected and diffused at its surface.

Hence, if the reflection, diffusion, transmission, and absorption of radiation be taken as established by experiment, we may, without further experiment, state that, if any medium exhibit any one or more of these properties in a marked degree, it must exhibit the others in a proportionately smaller degree.

It is now the function of experiment to analyse each of the above phenomena; in so doing it is necessary, in each case, to determine—

1. The general laws of the phenomenon.
2. The properties of different media in relation to the phenomenon.
3. The influence of the surface of the medium.
4. The influence of the nature of the source of heat from which radiation takes place.

We shall now briefly indicate the methods and results of experimental investigations carried out in this way.

183. Instruments used in experiments on radiation. Previous to the time of Melloni, the great difficulty in the experimental study of radiation lay in the fact that there was no instrument sufficiently sensitive to radiation to be of any use in accurate experiments.

Leslie obtained fairly satisfactory qualitative results with his **differential thermoscope**, described in Art. **62**. This instrument was largely used in the early days of the study of radiation.

The modern form of this instrument as adapted for the study of radiant heat is shown in Fig. 99. It comprises two bulbs connected by a bent capillary tube and containing ether and ether vapour. The lower bulb is coated with lampblack (see below). When radiant heat falls on the lower bulb its temperature rises and the vapour pressure of the ether is increased; as a consequence the ether rises in the tube leading to the upper bulb. The greater the quantity of radiant heat falling on the lower bulb the greater the rise of temperature, and consequently the higher the ether rises, and this height can be read off on a scale placed behind the capillary tube. Owing to external changes of temperature the quantitative results obtained with this instrument are not very reliable.

Fig. 99.

Modern experimenters have used the **thermopile**, which was first invented by Nobili, and afterwards improved by Melloni and others. This instrument consists of a number of short square rods of bismuth and antimony, arranged alternately side by side, so as to form a bundle having a square cross section. At the opposite ends of this bundle are the alternate junctions of the bismuth and antimony rods, so that if one end is heated a thermo-electric current is produced.

To employ this instrument as a thermometer it is connected up with a reflecting galvanometer, as shown in Fig. 100, and one end of the bundle of bismuth and antimony elements is then exposed to the radiation. The absorption of heat by this end produces a current which

deflects the galvanometer needle. Since the thermal effects involved are very small, this deflection may be taken as proportional to the heat absorbed by the exposed end of the pile. If this end be covered with a substance which absorbs an equal proportion of all kinds of radiation, then the galvanometer indication is proportional to the intensity of the incident radiation. We shall see later that lampblack absorbs every kind of radiation to the same degree. This substance is therefore employed to cover the ends of the thermopile.

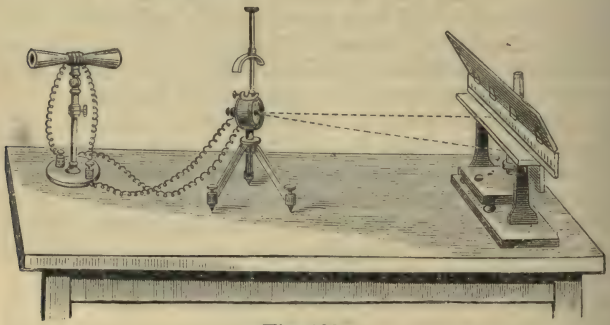


Fig. 100.

As a source of radiant heat we may use a cubical tin can containing boiling water. To increase the radiating or emissive power as much as possible one face of the can should be coated with lampblack. This cube is usually termed a **Leslie's cube**.

184. Propagation and transmission of radiation. The velocity of radiant heat and light through vacuo is the same, while the velocity of radiation of any particular wave-length through any body is inversely proportional to the refractive index of the body for that particular radiation.*

An experiment to show that radiant heat and light

* See *The Senior Sound and Light*: Light, Arts. **50**, **112**, and **116**.

travel at the same speed can be performed at the time of a total eclipse of the sun. Direct a thermopile to the position of the sun and wait until it emerges from behind the moon. Observe the galvanometer at the moment the sun shows itself. The needle is immediately deflected, showing that the heat and light take the same time to travel from the moon to the earth.

Exp. 119. To show that radiant heat travels in straight lines. Take two screens of wood A B and C D (Fig. 101) and bore a small hole (say ·5 cm. in diameter) through each. Arrange them parallel to each other, and opposite the hole in A B place the source of heat H with the blackened face turned towards A B.

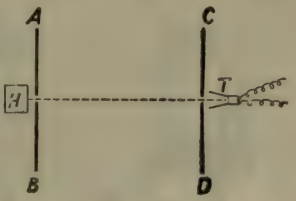


Fig. 101.

The thermopile T and its attachments are then placed on the far side of C D, the face of T being towards C D. T is now moved about. In general no deflection of the galvanometer will be observed. If, however, T be placed in the same straight line as the holes in the screens the galvanometer needle will be largely deflected.

185. The law of inverse squares. The law which governs the decrease of intensity of radiation with distance is the same as that for light, the two cases being identical.* The intensity at different distances from a point source of radiation is inversely proportional to the square of the distance. This may be proved experimentally by means of the arrangement shown in Fig. 102. A thermopile fitted with a reflecting cone is placed in front of a blackened large tin vessel (75 cm. × 75 cm. × 10 cm. is a convenient size) containing boiling water. If the thermopile is kept facing the vessel and moved about along a line perpendicular to the vessel, it is found that as long as the cone produced by the prolongation of the cone of the thermopile wholly cuts the front of the vessel the indication of the galvanometer remains unaltered.

* See *The Senior Sound and Light*: Light, Art. **14**.

Consider two positions, A and B, of the thermopile distant x and y respectively from the vessel. It follows by geometry that the areas of the circles from which radiation is received in the two cases are as x^2 to y^2. Since the galvanometer deflection remains unaltered, the influx of radia-

Fig. 102.

tion into the thermopile is the same in the two cases, and

$$\therefore \frac{\text{the intensity of the radiation at the position A from a small area of the vessel}}{\text{the intensity of the radiation at the position B from the same area of the vessel}} = \frac{y^2}{x^2},$$

i.e. the intensity of radiation at a point due to a small radiating area is inversely proportional to the square of the distance of the point from the area.

186. Reflection of radiant heat. The laws of reflection for radiant heat are the same as those for light. That is, the angle of incidence is equal to the angle of reflection, and both angles lie in the same plane. This fact is readily illustrated by experiment; in fact, any arrangement intended to exhibit the laws of reflection of light gives exactly similar results with radiant heat.

Exp. 120. Prove the equality of the angles of incidence and reflection. As a reflector use a flat polished tin-plate arranged as in Fig. 103. Place the source of heat H, a screen M R, and the plate P in position, and move the thermopile T about. In some positions the galvanometer needle will be reflected, showing that T is receiving heat. The screen M R stops direct transmission from H to T. Hence the only conclusion is that heat is reflected at P.

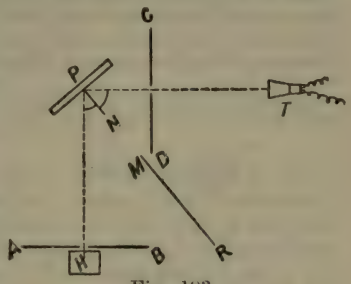


Fig. 103.

To show that the angle of incidence is equal to the angle of reflection, take the two screens mentioned above, and place A B in such a position that the incident ray H P is inclined about 45° to the plate P. Move C D and T about till the galvanometer deflection is a maximum. Join up H P and T P, draw the normal P N and show that the angles H P N and T P N are equal.

The following is another common experiment:—

Exp. 121. Set up two large concave mirrors, M, M′ (Fig. 104), facing each other, and about 3 feet apart, with their axes in the same

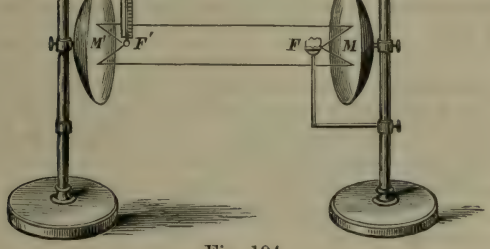


Fig. 104.

straight line. At the principal focus, F′, of one mirror, say, M′, support the bulb of a small air thermometer. At the principal focus F of the mirror M place an iron ball heated to a red heat.

The thermometer shows a rapid rise of temperature as long as the bulb is kept at F', but, if moved into any other position, the temperature immediately falls, thus showing that the radiation from the iron at F is first reflected from M on to M' and then converged to F'. The result is not affected by placing a small screen between F and F' to prevent the thermometer being influenced by direct radiation from the iron.

Vary the experiment by placing a piece of phosphorus at F'. If the iron ball is sufficiently hot, the phosphorus quickly takes fire.

187. Reflecting power. The *reflecting power* of a surface is measured by the ratio of the amount of regularly reflected radiation to that of the incident radiation. For example, if the radiation incident on a surface be denoted by R, and the quantity reflected by Rr, where r is less than unity, then the reflecting power is measured by the ratio

$$\frac{Rr}{R} = r.$$

Relative reflecting powers may be measured by placing various reflectors in turn at P (Fig. 103) and noting the indications of the galvanometer attached to the thermopile T. The reflecting power of a surface of course increases with the degree of polish of the surface, and it also varies with the nature of the source of radiant heat (*i.e.* it depends on the quality of the radiation).

Polished metals are the best reflectors. Roasting screens, firemen's helmets, and the reflectors of gas-stoves are familiar applications of the principles governing the reflection of heat.

188. Diffusion of radiant heat. When radiation is incident on a rough or unpolished surface, it is, as it were, broken up by the inequalities of the surface, and undergoes irregular reflection or diffusion.

The **diffusive power** of a surface is determined by the ratio of total quantity of diffused radiation to the quantity of incident radiation.

The existence of diffusion may be shown by means of the apparatus of Fig. 103. The roughened surface to be experimented on is placed at P, and the thermopile adjusted in the position T. In this and similar positions

the thermopile indicates *at once* the absorption of radiation by the face turned towards P. This radiation cannot be *reflected* from the surface at P, for that surface is not polished, and the effect may be observed for *all positions* of P in front of the plane of the surface. The effect can therefore be due only to diffusion, or to radiation from the surface heated by direct radiation from H, but since the readings are observed instantaneously, *i.e.* before P has time to be heated, none of the effect can be due to radiation from P.

The diffusive power of a surface varies with the source of heat and with the nature and material of the surface. In general a rough, uneven surface, such as that of a fine powder, diffuses best; but this depends upon the *absorbing* power of the material of the surface. For example, a rough lampblack surface would *absorb* more than 90 per cent. of the incident radiation, and diffuse less than 10 per cent.

189. Refraction of radiant heat. The laws of the refraction of radiant heat are identical with those of optical refraction. This can be shown by the following experiment.

Exp. 122. To show that radiant heat obeys the laws of refraction. Arrange the usual apparatus for obtaining a pure spectrum,* using a strip of platinum heated to incandescence by a Bunsen flame or an electric current as the source of light and heat. Adjust the face of a thermopile so as to receive the portion of the spectrum lying below the red; the galvanometer at once indicates the absorption of heat. Now rotate the prism so as to displace the spectrum some distance on either side of the pile; the galvanometer deflection quickly falls to zero.

Again place the pile in the same position as before, relative to the visible spectrum; the deflection will again increase to something near its original value. Again, with the thermopile in position, allow the platinum to cool slowly; the deflection shows a slight continuous decrease as the visible spectrum gradually fades out from the violet to the red end; but soon after this point is reached a much more marked decrease is observed, indicating that the radiation incident on the pile is, in its turn, disappearing.

Experiment 122 and others of a similar character show that both the luminous and non-luminous radiation from

* See *The Senior Sound and Light*: Light, Art. 95.

an incandescent source of heat are refracted according to the same general laws, and also that the *refrangibility* * of the non-luminous portion of the radiation is lower than that of the luminous portion. *Rock-salt* prisms and lenses should be used in the experiments, for glass absorbs a large percentage of the non-luminous radiation, while rock-salt absorbs less than 10 per cent. of it. (See Art. 190.)

190. Measurement of diathermancy. The proportion of the incident radiation transmitted by a given thickness of a substance is a measure of the *power of transmission* or *diathermancy* of that thickness of the substance for the radiation considered.

Exp. 123. Determine the diathermancy of different substances. Set up apparatus as indicated in Fig. 105.

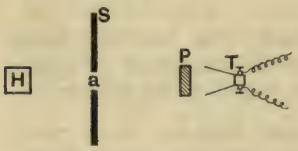


Fig. 105.

H is the source of heat, S a screen, with a circular aperture, *a*, through which radiation from H falls on P, the plate of the substance to be tested. Place the thermopile at T and note the galvanometer deflection before and after interposing the plate P. The ratio of these deflections determines the diathermancy of the substance for the thickness considered. Repeat with plates of other substances of the same thickness.

In order to determine the diathermancies of liquids, they are placed in cells of thin glass and indications taken first through the empty cell and then through the cell filled with liquid. Gases are studied in the same way. In all cases except rock-salt the diathermancy depends on the temperature of the source of radiation.

The following facts, determined by experiment, are worth noting:—

1. Rock-salt is the most diathermanous substance. It transmits about 90 per cent. of the incident heat.

* See *The Senior Sound and Light*: Light, Art. 94.

2. Glass transmits about 50 per cent. of the radiation from a source of heat at a high temperature (*e.g.* solar radiation), but is almost adiathermanous to radiation from a source at a temperature below a red heat.

3. Alum in crystalline plates is similar to glass, but is even more opaque to *dark heat.*

4. Water is fairly adiathermanous. It only transmits 10 per cent. of the heat radiation. Although alum is more adiathermanous than water, yet the addition of alum to water raises the diathermancy of the latter.

5. A solution of iodine in carbon disulphide is opaque to visible rays, but is particularly transparent to the dark heat rays.

These facts are sometimes applied to determine roughly the nature of radiation. For example, if the radiation considered is incapable of transmission by an alum crystal, then it must consist entirely of dark heat rays.

Exp. 124. To compare the diathermancies of water and carbon bisulphide. Focus the sunlight upon the bulb of an ordinary thermometer, and place between the bulb and the lens a flat glass cell filled with water. Observe the reading of the thermometer, and then replace the cell by a similar one filled with bisulphide of carbon. Note that the temperature indicated by the thermometer is higher in the second case, thus showing that the bisulphide is more diathermanous than water. If a little iodine be now dissolved in the bisulphide, it becomes opaque to light, but is still very diathermanous, thus proving that a body which is non-transparent to light may be transparent to heat.

191. Influence of thickness on transmission and absorption. Experiments have shown that when radiation of a definite kind is transmitted through a substance the amount transmitted decreases in geometrical progression as the thickness increases in arithmetical progression—that is, each layer of the substance, of a given thickness, transmits the same proportion of the radiation which enters it.

Thus, if q denote the quantity of radiation entering the substance, then, after traversing unit thickness, it becomes qa where a is a constant. Similarly, on transmission through another layer of unit thickness, it is

reduced to $q\,a^2$; and therefore, after transmission through a thickness, n, the quantity of transmitted radiation is given by $q\,a^n$. The constant a has been called the *coefficient of transmission.*

When radiation of a compound nature is transmitted by any substance, its various constituents are absorbed to different degrees, and thus the nature of the transmitted radiation is subject to continuous change. The character of this change is, however, such that the nature of the transmitted radiation tends to become constant and capable of transmission without absorption. For this reason radiation which has passed through a plate of any substance passes readily with little loss through another plate of the same substance.

Example. A plate of rock-salt 2·5 mm. thick allows 93 % of the incident radiant heat to pass through it. How much will a plate four times as thick transmit? Assume that there is no loss by reflection at the surfaces. Find also the coefficient of transmission of rock-salt.

Then 93 % gets through one plate,
93 % of 93 % ,, ,, a plate twice as thick,
93 % of 93 % of 93 % ,, ,, a plate three times as thick,
93 % of 93 % of 93 % of 93 % ,, ,, a plate four times as thick, *i.e.* 1 cm. in thickness.

This is equal to about 75 %, *i.e.* therefore the coefficient of transmission is ·75.

192. Newton's law of cooling. Consider a body placed in an enclosure at a constant temperature slightly lower than that of the body. If the difference of temperature is small enough, we may assume that the rate at which the body loses heat is proportional to this difference. Hence, if S denote the area of the surface of a body, θ the excess of its temperature above that of the enclosure, and E a constant varying with the nature of its surface and the state of the surrounding medium, then the heat lost in a small interval of time, t, is given by

$$h = \mathrm{E}\,\mathrm{S}\,\theta\,t.$$

Again, if the body be a good conductor of heat, so that

its temperature is the same at all points, this loss of heat may be expressed by

$$h = m\,s\,\tau,$$

where m denotes the mass of the body, s its specific heat, and τ the small change of temperature accompanying the loss of heat. Thus we have

$$\mathrm{E\,S}\,\theta\,t = m\,s\,\tau,$$

or

$$\frac{\tau}{t} = \frac{\mathrm{E\,S}}{m\,s}\,\theta.$$

That is, *the rate of cooling* $\left(\frac{\tau}{t}\right)$ *is directly proportional to the excess of the temperature of the body over that of the enclosure.* This is Newton's law of cooling. E is called the **coefficient of emission** or **emissive power**.

Exp. 125. Verify Newton's law of cooling. Place some hot water in a copper calorimeter blackened on the outside and arrange matters as in **Exp. 34**. Take readings every 5° C. as the temperatures cool from, say, 80° C. to about 35° C. Plot a curve between temperature and time, and show from it that the rate of cooling is very nearly proportional to the difference of temperature between the water in the calorimeter and the surroundings.

If the temperature of the body is below that of the enclosure a similar formula

$$\frac{\tau}{t} = \frac{\mathrm{A\,S}}{m\,s}\,\theta$$

may be deduced where A is a constant of like nature to E, and is called the **coefficient of absorption** or **absorptive power**. It will be proved later that for a given temperature of the body and a fixed value of θ the coefficients E and A have the same numerical value.

193. Comparison of emissive powers. The determination of the *comparative emissive powers* of different surfaces is a simpler matter than determining the coefficient of emission for a given substance.

Leslie employed as source of heat a thin metal cube containing boiling water. The sides of this cube were

covered with substances whose emissive powers were to be tested. The radiation from one face of the cube was concentrated by a concave mirror on to one of the bulbs of a differential thermoscope. The indication being noted, the other faces of the cube were tested in the same way, and the relative emissive powers were taken as proportional to the corresponding indications of the thermoscope.

Melloni, adopting the same method, employed the thermopile instead of the differential thermoscope.

Exp. 126. To compare the emissive powers of different surfaces. Take a cubical tin canister, cover one vertical face with lampblack, another with white lead, a third with paper, and polish the fourth. Fill it with water and keep it boiling by a protected flame. Bring the face of the thermopile or the blackened * bulb of an air thermoscope to the same distance (one inch) from each side in turn, and for the same time (three minutes). Note that the index moves farthest when the bulb is opposite the lampblack, and least when opposite the polished tin. Other surfaces may be compared in the same way.

The following are some relative emissive powers of surfaces (when kept at 120° C.):—

Lampblack	...	100	Platinum (polished)		10
White lead	...	100	Copper foil	...	5
Paper ...	...	98	Silver (polished)		2

Exp. 127. *Obtain the cooling curves for water in a tin can*, whose outside is (i) polished, (ii) jacketed with cotton wool, (iii) coated with lampblack (hold it in the smoky flame of a lamp, or of burning camphor), (iv) when the blackened can is suspended by cotton loops within a larger one whose inside is bright, (v) as (iv) but when the inside of the larger can is blackened. Use the same quantity of water in each experiment, begin observations when its temperature is about 80° C., and continue to about 40° C.

Tabulate each case and *plot* all to a large scale on one sheet of paper. The curves will show roughly the relative *emissive powers* of the surfaces when (1) polished, (2) lagged, *i.e.* jacketed with a non-conductor, (3) blackened, (4) when screened by a reflecting surface, (5) when screened by a blackened surface.

194. Absorption. The coefficient of absorption has been defined above in connection with the coefficient of emission. The relation between these two coefficients seems to indicate

* The bulb is blackened in order to increase its absorbing power.

that the absorbing power of any surface is proportional to its emissive power for the same kind of radiation. Experiment has shown that this is so.

The determination of absolute absorbing power, as defined above, is somewhat difficult, but the comparison of absorbing powers is more easily effected.

The arrangement of apparatus employed by Melloni for this purpose is indicated in Fig. 106. H represents the source of heat, S a perforated screen, T the thermopile, and C a sheet of very thin copper plate coated on the front surface with the substance to be tested, and on the surface next the pile with lampblack. The first substance tested was lampblack, which, as in the case of emission, was taken as the standard substance. The relative absorbing powers of different substances were then deduced from the data of the experiments by a formula which is easily obtained.

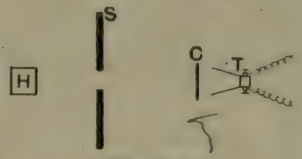


Fig. 106.

The following are some relative absorbing powers :—

Lampblack	...	100	Shellac ...	...	72
White lead	...	100	Metals ...	...	12

It will be seen that these results are practically identical with those given in Art. **193** for the relative emissive powers. The absorbing power, like the emissive power, also depends on the temperature of the source.

It should be noticed that absorption differs from the other phenomena considered in this chapter in that, after absorption, the radiation ceases to exist as radiation, and becomes ordinary heat due to molecular motion. Hence, when a body *absorbs* radiation it rises in temperature, whereas a body which transmits radiation does not rise in temperature.

When an adiathermanous body absorbs heat, it is probable that the absorption takes place at the surface or in a very thin superficial layer ; but in a diathermanous body absorption takes place during transmission, and, it may be, at the surface also. Lampblack is the only one of the substances

which have been tested which does not exhibit selective absorption. It absorbs all kinds of radiation to the same degree.

195. Theoretical relations between reflection, diffusion, absorption, and emission. Let Q denote the quantity of radiation incident on the surface of a body. This, in the most general case, may be divided into two parts: Q′, which is arrested at the surface, and q, which is transmitted by the substance of the body. That is, $Q = Q' + q$. Now, considering Q′, we have, as in Art. **182**,

$$Q' = Q(r + d + a).$$

The quantity q is diminished by continuous absorption during transmission; and, if a denote the coefficient of transmission (Art. **191**), then $q\,a^n$ gives the intensity of the transmitted radiation after passing through a thickness n cm. of the substance.

We shall now consider some particular cases of this general case. In each case the substance is supposed to be adiathermanous, so that $q = 0$, and therefore $Q' = Q$.

1. **Let the surface be polished.** If the polish is sufficiently perfect, $d = 0$. Hence, we have

$$Q = Q(r + a).$$

$$\therefore 1 = r + a,$$

or—

$$a = 1 - r.$$

That is, the absorbing power of the surface is complementary to its reflecting power. Hence, when a reflecting surface of this nature reflects well it absorbs badly—that is, when r is large a is small.

2. **Let the surface be rough.** The surface being unpolished, $r = 0$. Therefore

$$Q = Q(d + a).$$

$$\therefore a = 1 - d.$$

That is, the absorbing power is complementary to the diffusive power. Hence, when a surface diffuses well it absorbs badly.

From these two cases it is evident that, under the conditions considered, a can be calculated when r or d is known.

3. **Let the surface be imperfectly polished.** Then we have

$$Q = Q(r + d + a).$$
$$\therefore a = 1 - r - d = 1 - (r + d).$$

That is, the absorbing power is complementary to the sum of the reflecting and diffusive powers. By polishing the surface, r is increased and d decreased, so that the sum $(r + d)$ may remain constant, increase, or diminish. That is, the absorbing power of a surface may remain constant for different conditions of polish, or may increase or diminish with the degree of polish.

If the diathermancy of a plate of rock-salt is tested *for radiation* from a hot piece of the same substance, it will be found to be very small; a thin plate absorbs more than 75 per cent. of the radiation; and, if the plate be at the same temperature as the radiating piece, it will be found that none of the incident radiation is transmitted by it. Hence the radiation absorbed by any substance at a given temperature is of the same nature as that emitted by the substance at the same temperature.* From this it follows that **the absorbing power of any substance is proportional to its emissive or radiating power at the same temperature.**

The colour of a body has great influence on its absorbing power: *e.g.* a white coat absorbs less heat in the sunshine than a black one.

Exp. 128. Take two equal polished tins (*e.g.* cigarette tins, coffee tins, etc.); blacken the surface of one by means of a lamp flame. Fill both with water, and set them in front of a bright fire. After half an hour test the temperature of the water with a thermometer. The water in the blackened tin will be the hotter.

---

* Compare this with the production of the Fraunhofer lines in the solar spectrum. (See *The Senior Sound and Light*: Light, Chap. xiii.)

196. Prevost's Theory of Exchanges. *This theory asserts that every body emits radiation in all directions to an extent determined only by its temperature and the nature of its surface, and that it absorbs radiation from all sources to which it is exposed.*

Suppose that in Exp. **121** a piece of ice takes the place of the hot iron ball at F (Fig. 104). It will be found that the thermometer at F′ indicates a fall of temperature. This result is very neatly explained by the above theory. The ice at F radiates in all directions at a rate dependent on its temperature. A portion of this radiation is reflected by the mirror M on to M′, and thence to the bulb of the thermometer at F′. Similarly, the thermometer radiates in all directions at a rate dependent on its temperature, and a portion of this radiation is, after reflection by M′ and M, concentrated on the ice at F. Hence, considering the thermometer, it gains heat by radiation from the ice, and it loses heat by its own radiation; and in this case, since the temperature of the thermometer is higher than that of the ice, the loss is greater than the gain, and the temperature falls. In Exp. **121** with the hot iron the gain is greater than the loss, and the temperature of the thermometer rises.

When bodies are at the same temperature, each loses as much by its own radiation as it gains by radiation from surrounding bodies, and thus the temperature of each remains constant. Consider an enclosure maintained at a constant temperature, and having a thermometer suspended in its interior. Whatever may have been the initial temperature of the thermometer, it ultimately takes and retains the constant temperature of the enclosure. When this is the case the heat which the thermometer gains by radiation from the walls of the enclosure equals the heat which it loses by its own radiation. That is, the radiation absorbed by any body at a given temperature is equal and identical with the radiation emitted by the same body at the same temperature. This result we have already deduced from experiment, and it therefore affords an indirect experimental proof of the theory of exchanges.

Again, if a large number of thermometers, whose surfaces are covered with different substances, be suspended in the enclosure, they all ultimately take up a common constant temperature equal to that of the enclosure.

Now, considering any one of these thermometers, we have evidently a more complex system to deal with than in the case just considered. The thermometer receives radiation from the walls of the enclosure, and also from each of the thermometers. The radiation from the walls is of a definite kind, determined by the nature of the surface. Also the radiation from each of the thermometers is of a definite kind, similarly determined; but the nature of the surface being different in each case the radiation emitted is also different. Hence, the thermometer considered receives a large number of different kinds of radiation, and emits one kind of radiation determined by the nature of its own surface. But since the temperature remains constant it follows that heat gained must equal the heat lost—that is, the radiation emitted must be identical in quantity and *quality* with that absorbed.

This statement seems to imply that the radiation of any body is determined by the radiation it absorbs, and not by any inherent property. This is, of course, not the case; the radiation emitted by any body cannot be controlled by any merely external circumstances. If the thermometer emits radiation other than that which it absorbs—that is, *other than that which is emitted by the bodies from which it receives radiation*—then this radiation, since it is not emitted by the other bodies, cannot be absorbed by them, and is therefore returned to the thermometer by reflection or diffusion, and is there absorbed. Thus, it is evident that in an enclosure of this nature the stream of radiant heat is constant in quantity and quality, for each surface restores by its own radiation a quantity exactly equal and similar to that which it absorbs.

For example, imagine four of the thermometers to have respectively surfaces of lampblack, gold-leaf, glass, and powdered silver. The lampblack surface absorbs nearly all the incident radiation, and emits a corresponding quantity.

The gold-leaf surface reflects a large percentage and absorbs the remainder; hence, it emits but a small amount. The glass surface reflects and transmits a proportion depending on the nature of the radiation, and absorbs the rest, which corresponds to the radiation which it emits. The powdered silver surface diffuses a large amount, and therefore absorbs and emits but little.

From what has been said above it will be seen that for any two bodies, A and B, exposed to each other's radiation at the same temperature, there is a *constant, interchangeable* quantity of radiation, capable of absorption by each. Thus, of the radiation emitted by A, a portion, q, of fixed quantity and quality, is absorbed by B; this is again emitted by B, forming a portion of the entire radiation emitted by that body.

When this radiation from B falls upon A, the quantity q only is absorbed, for every constituent capable of emission by B and absorption by A is also capable of emission by A and absorption by B, and is therefore included in the quantity q. Hence if we denote the radiation emitted by A by $m\,q$, and that emitted by B by $n\,q$, we have

$$\frac{\text{Radiating power of A}}{\text{Radiating power of B}} = \frac{m}{n}.$$

Also—

$$\frac{\text{Absorbing power of A for radiation emitted by B}}{\text{Absorbing power of B for radiation emitted by A}} = \frac{\frac{q}{nq}}{\frac{q}{mq}} = \frac{m}{n}.$$

That is, the absorbing power of any substance, A, for radiation emitted by any other substance, B, at the same temperature, is to the absorbing power of B for radiation from A as the radiating power of A is to that of B.

For example, consider the case of a thermometer suspended in an enclosure at constant temperature. Suppose the radiating power of the walls of the enclosure to be five times that of the thermometer, and that the walls absorb one-fourth of the radiation from the thermometer; then, in order that the temperature of the thermometer may remain constant, it must absorb a quantity of heat equal

to one-fourth of its own radiation—that is, to one-twentieth of the radiation from the walls of the enclosure which, by hypothesis, is five times that from the thermometer. Hence, the radiating and absorbing powers of the thermometer and the walls of the enclosure have the common ratio 1 : 5. This relation is true for each constituent of the radiation to which it applies.

197. Radiation of gases. Gases in general radiate badly, but in most cases the phenomena of selective absorption and radiation are very strongly marked.

The radiation from a solid, at a given temperature, is generally included within a certain range of wave-length. As the temperature increases this range is extended in the direction of increasing refrangibility, while each constituent increases in intensity.

Gases, on the other hand, emit radiation made up of one or more constituents of definite wave-length; as the temperature increases other constituents of shorter wave-length may be added, and the intensity of each of the original constituents is increased.

198. Illustrative experiments and phenomena. We shall here consider some natural phenomena and a few more simple experiments which illustrate the principles and laws deduced in this chapter.

The effect of the surface of a body on its emissive or radiating power may be seen by noting the rate of cooling of water in a vessel the surface of which can be varied. If the vessel has a brightly polished metallic surface the water cools very slowly, but if covered with lampblack the rate of cooling is very much faster. For this reason teapots and vessels intended to keep their contents warm should have brightly polished surfaces. Similarly, if a vessel having a brightly polished surface is exposed to radiation it absorbs heat very slowly, whereas if the surface is rough and blackened the incident radiation is more rapidly absorbed. The exterior of a calorimeter is kept bright so that it may neither radiate nor absorb heat. Hot water pipes are blackened to increase the radiation from them. These experiments show that a good reflecting surface radiates badly and also absorbs badly. Lampblack is the best radiator and absorber of heat known, but it is the worst reflector.

These results may be exhibited with the apparatus shown in Fig. 107. A is a sheet of thin copper with a brightly polished surface facing B, another similar sheet, having a lampblacked surface turned towards A. A and B are connected by a copper wire, *c c*. Two iron wires *i* and *i'* are joined, as shown in the figure, to the backs of A and B, and to the galvanometer G. A copper-iron electric circuit* is thus formed, having the junctions on A and B. A heated ball of metal M is now supported midway between A and B, so that the copper plates are equally exposed to its radiation. Before long the galvanometer shows a gradually increasing deflection, indicating that the junction at B is heating more rapidly than that at A.

Fig. 107.

The following experiment illustrating the same point is due to **Tyndall**.

Exp. 129. Write a word with ordinary ink on a piece of bright thin platinum foil and then heat the foil to a bright white heat. It will be seen that the written characters which have a rough surface of iron oxide glow much more brightly than the rest of the foil. If the reverse side, on which there is no writing, is observed it will be seen that the writing is visible on it as a dark pattern on a bright ground. The letters, on account of the nature of their surface, radiate more than the foil, and therefore cool more rapidly. Hence the foil immediately underlying the writing also cools by conduction more rapidly than the rest of the foil, and thus appears dark in comparison with the other portions of the same surface.

Exp. 130. Obtain a piece of white earthenware bearing a dark coloured design. When cold the design absorbs most of the light falling on it, the rest of the earthenware appears white by diffusion. Now heat the earthenware in a fire or gas furnace. When quite hot withdraw it and place it in the dark. Observe now that the design gives off more light than the other portion of the earthenware.

Ritchie devised a very convenient form of apparatus to show the equality of the radiating and absorbing powers

* See Art. **183** for some of the principles of the thermo-electric circuit. For further information see *The Senior Magnetism and Electricity*: Art. **228**.

of a surface. It is shown in Fig. 108, and comprises two cylindrical reservoirs full of air joined together by a glass tube bent twice at right angles and carried on a stand at the back of the two upright glass tubes. This portion constitutes a Leslie's differential thermoscope.

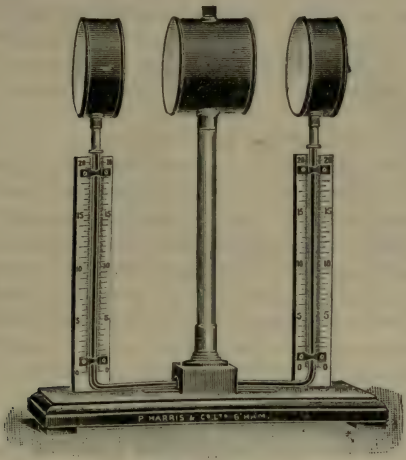


Fig. 108.

Between the two reservoirs is a third and larger one which can be filled with hot water. It is movable along a horizontal support. Each small reservoir has one face coated with lampblack, the other being left bright. The black end of one and the bright face of the other face inward. The centre reservoir has similarly one face black and the other polished, and it is placed so that its bright end faces a black end of one small reservoir and its black end the bright end of the other small reservoir.

When the centre reservoir is filled with hot water its bright face radiates to a black face, and its black face to a bright face. It is found by experiment that when the centre reservoir is midway between the other two the

liquid in the stem of the thermoscope does not move, indicating that the air in the two reservoirs is at the same temperature.

Gases are poor radiators of heat, hence in the construction of furnaces the hot gases are made to play on some substances such as fire-bricks, which are good radiators. These become heated by direct contact with the hot gases and then radiate the heat freely in all directions. Asbestos balls used in gas-fires and fire-clay bricks used as backs to ordinary grates further illustrate this principle.

The rare oxides thoria and ceria when hot emit light very readily; hence their use in the Welsbach mantle.

The various phenomena of **diathermancy** are capable of simple illustration. If radiation is transmitted through a plate of ice, the interior of the plate begins to melt, showing beautiful hexagonal ice crystals; but the radiation, after passing through this plate, may be passed through another plate or lens of ice without causing the least melting. This shows that the absorption which radiation undergoes during transmission through any substance is such as to fit it for further transmission without absorption.

The **glass fire-screens** occasionally used are applications of the fact that the diathermancy of glass varies with the nature of the radiation. The radiation from the fire falling on the screen is partly absorbed and partly transmitted; the more refrangible luminous portion is transmitted, while the less refrangible portion, of greater heating power, is absorbed and afterwards radiated by the screen. In this way the cheerful firelight is transmitted, but the more intense heat rays are absorbed, and afterwards emitted by the screen as radiation of a less intense nature.

Glass is employed in **hot-houses** for the following reason. Of the solar radiation incident on the glass only the more refrangible luminous portion is transmitted. This portion, however, forms a large percentage of solar radiation, and has considerable heating effect; hence, when absorbed by the objects within the hot-house, it gradually raises their temperature. The radiation from these objects, being of low refrangibility, is not transmitted by the glass, and

cannot in this way escape to the exterior. This radiation is, however, absorbed by the glass; but, owing to absorption of a portion of the solar radiation, the glass is at a higher temperature than the objects inside the house, and therefore returns to them by its own radiation more than it receives. In this way the temperature of the interior gradually rises, and will continue to do so until it attains the temperature of the glass.*

Water vapour in the atmosphere acts in much the same way towards the earth as the glass does towards the objects inside a glass house. Dry air is very diathermanous to radiant heat, but the absorptive power of air charged with water vapour is from 30 to 70 times as great as that of dry air, while it is still very transparent to rays of light. Thus the earth is protected by the absorptive action of the water vapour from the more intense heat rays of the sun; but when the warmed earth begins to radiate its heat in rays of low refrangibility these rays find a most effectual barrier to their escape in the water vapour, which absorbs them freely and becomes warmed thereby.

In this way extremes of temperature are avoided—the direct heating effect of the sun during the day and the loss of heat by radiation during the night are both diminished.

In districts where little water vapour is present in the air—for example, in the Desert of Sahara—the days are intensely hot and the nights extremely cold. Probably other causes have something to do with these and similar results, but the action of water vapour has undoubtedly an important bearing on the question.

In summer the plains and valleys especially are covered by a protecting stratum of vapour which serves to prevent the escape of radiant heat, and when night falls they remain comparatively warm. But sandy deserts, mountain

* The above explanation has recently been controverted by R. W. Wood, who says that the only action of the glass is to keep the same air within the house all the time, and the temperature of this air will, of course, naturally rise as the sunshine continues.

tops, and elevated table-lands, above which there is much less vapour, cool down quickly.

A **vacuum** is a perfect non-conductor of heat. Double-walled vacuum vessels of glass are now made of the forms shown in Fig. 109. They consist of a flask or tube inside another, the two being sealed together at the top and the space between them exhausted of air as completely as possible through a fine tube at the bottom of the outer vessel, which is then sealed off. As a rule the surfaces of the vessels next the vacuum are silvered to prevent loss or gain of heat by radiation. These vessels were first designed for the storage of liquefied gases, which would boil away at once if placed in ordinary vessels. Liquid air may be kept for days in a good vessel. They are now also largely used in the form of "**Thermos**" flasks for keeping hot drinks hot and cold drinks cold for a long time.

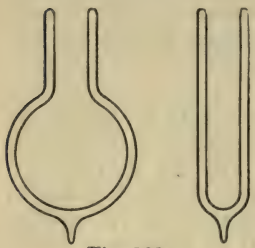


Fig. 109.

EXERCISES XVIII.

1. What is the meaning of radiant heat? In what way does it differ from light?

2. Describe an experiment to show that radiant heat obeys the same laws as light.

3. Should a kettle intended to be heated by standing in front of a fire be bright or black? State fully the reasons for your answer.

4. A thermometer placed in an open black box is exposed to the sun; when a glass cover is placed on the box the temperature of the thermometer rises. Explain the cause of this rise of temperature.

5. Describe an experiment to measure the relative diathermancies of two bodies.

6. If it were required to keep the temperature of a jug of water for as long as possible below the temperature of the room, would it be better to use a metal or an earthenware jug?

7. What methods have been used to measure the radiation from heated bodies, and what laws have been discovered?

8. What experimental evidence have we that bodies absorb when cold the same kind of radiation they give out when hot?

9. A clear piece of rock-salt, a bright piece of tin-plate, and a piece of black wood are exposed to the radiation of a common fire. State what happens in each case, and what is the final state of each as regards temperature.

10. Define the coefficient of transmission of a substance for radiation. If the coefficient of absorption of a substance for a particular variety of radiation is ·0012, what percentage of the incident radiation will get through a plate of the substance 3 mm. thick?

11. If a plate of glass allowed 40% of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming that no light is lost by reflection at the surfaces in either case?

12. Letters are written in ink upon a bright piece of platinum-foil which is subsequently placed in the flame of a Bunsen burner in a dark room. Looked at on one side bright letters appear on a dark ground; on the other side, dark letters on a bright ground. Explain this.

13. Describe experiments which show that the absorbing power of a surface is proportional to its radiating power for any, the same, kind of radiation.

14. If the bulb of a blackened thermometer be enclosed in an outer case of glass, a vacuum being between the two, it is seen that when exposed to the sun's rays its temperature becomes high. Explain this, and mention any transparent substance which would not have the same property in this respect as glass.

15. What special properties in relation to radiation are shown by lampblack, brightly polished silver, glass, quartz, rock-salt, alum, solution of iodine in carbon bisulphide?

EXAMINATION QUESTIONS.

1. Define the conductivity of a substance for heat. How would you propose to find whether a liquid was able to conduct heat or not?

2. Define the *thermal conductivity* of a substance.
What is meant by the statement that the thermal conductivity of glass is ·001 in C.G.S. units?

If the inside surface of a glass window 5 millimetres thick is at a temperature of 15° C., and the outside surface is at a temperature of 0° C., how much heat escapes through a square centimetre of the window in a minute?

3. It is found that the temperature of the rocks below the earth's surface increases at the rate of (approximately) one degree C. for each 30 metres of descent. Taking the thermal conductivity of the rocks as ·005 in the C.G.S. system, calculate how much water would be raised from the freezing point to the boiling point by the heat arriving in the course of a year at a square metre of the earth's surface from its interior.

4. One end of a long rod is kept at 100° C., the remainder of the rod being exposed to the air. Will the temperature of the rod at a given distance from the hot end be the same whether the rod is made of copper or of wood? Give reasons.

In the first case, will the temperature from point to point vary in the same way whether the surface of the copper be polished or blackened? How would you find the temperature at different points of the rod?

5. How would you determine the relative conductivities of two bars, equal in length, shape, and area, of cross section, but of different metals?

6. How is heat transferred from one part of a liquid to another? Describe a few simple experiments to illustrate your answer.

7. The ends of similarly dimensioned rods of iron and lead were soldered externally to the sides of a vessel; shots were affixed to the lower sides of the rods by means of soft wax. Then boiling water was poured into the vessel and maintained at the boiling point. Describe and explain what may happen.

8. What is implied in the statement that the conductivity of iron is ·152 C.G.S. units?

What would be the loss of heat per square metre per minute from the walls of a house which is kept internally at 15° C., the external temperature being 0° C.? The conductivity of the material of the walls is ·005, and they are 25 cm. in thickness.

9. A rod of metal is heated at one end. Upon what does the rate of rise of temperature at any point in the bar depend?

Describe an experiment to illustrate your answer.

10. Distinguish between radiation, convection, and conduction of heat.

11. What do you understand by the term *radiant heat*? Can you suggest a better name for it?

Describe experiments to show (1) that radiant heat travels in straight lines, (2) that it is reflected, and (3) that it is refracted according to laws identical in form with those regulating the reflexion and refraction of light.

How might an observation be made at a total eclipse of the sun to suggest that radiant heat travels with the speed of light?

12. Describe an experiment to show that different surfaces may have different absorbing powers.

13. A, B, and C are three brightly polished tinned cans of the same size. A is covered with a tightly fitting jacket made of a single layer of thin flannel. B is jacketed with a dozen layers of the same flannel. C remains uncovered. The same quantity of boiling water is poured into each can, and the times required for the water to cool from 90°—80° C. are noted in each case. B's time is found to be the longest and A's time the shortest. Explain these observations.

14. State Newton's law of cooling of a hot body.

How would you determine the specific heat of a liquid by making use of this law?

15. Describe, giving reasons, the kind of receptacle you would use for preserving ice for as long a time as possible in a warm room.

16. A bright tin vessel and an equal blackened tin vessel, containing equal quantities of water, are placed side by side in front of a hot fire. Compare the way in which the temperature will rise in the two.

If they are filled with boiling water and placed in the middle of a room, compare the way in which the temperature will fall in the two. What general law do the experiments illustrate?

17. A person who glazes his greenhouse with glass with a view of keeping his plants warm, uses a glass screen to keep off the heat of the fire. Give reasons which justify his practice.

18. Give a short account of Prevost's Theory of Exchanges. A lampblacked ball of wood suspended inside a sphere of polished silver kept at a constant temperature will reach and maintain the temperature of the enclosure. Basing your argument on this fact, show that it necessarily follows from Prevost's theory that poor emitters of radiation must be correspondingly poor absorbers of radiation.

19. A large slab covered with lampblack is kept constantly at 100° C. The effect it produces on a thermopile furnished with a dead-black cone does not alter as the perpendicular distance of the pile from the centre of the slab is varied. Describe fully what inference may be drawn from this fact.

20. Heat was at one time supposed to be a substance called caloric; what experiments would you bring forward to show that this view is untenable?

21. What is meant by the mechanical equivalent of heat? Describe an experiment to show that the temperature of a gas is raised when it is compressed suddenly.

22. What reasons have we for believing that heat is a form of energy?

From what height must a piece of ice at 0° C. fall in order to melt itself, assuming that all the energy of the fall remains in the ice?

23. Lead shot weighing 500 gram. and at a temperature of 10° C. are placed in a long asbestos tube closed at the ends. The tube is first held vertically, and is then suddenly turned completely over so that the shot fall down the tube. This turning movement is repeated quickly 100 times, and the final temperature of the shot is found to be 18° C. Assuming that the average distance through which the shot fall at each inversion of the tube is one metre, calculate the mechanical equivalent of heat. State the units in which your answer is expressed. [Specific heat of lead = ·03.]

24. Explain why it is that while the numerical value of the latent heat of water is less when expressed in terms of the Centigrade scale than when expressed in Fahrenheit's scale, just the opposite holds in the case of the numerical values of the mechanical equivalent of heat.

25. Describe fully what is implied in the statement: The value in ergs of Joule's equivalent is $4{\cdot}2 \times 10^7$.

The churn of a Joule's apparatus contained 10 kilograms of water, and a man by turning the paddle for one hour caused the temperature of the water to rise 6·42° C. At what rate did the man work? (Give your answer in absolute units.)

26. Calculate the velocity of a lead bullet on striking an unyielding target, if its temperature rises 200° C. and the whole of the heat generated by the impact remains in the lead. [Specific heat of lead = ·03.]

27. Calculate the quantity of heat absorbed by a perfect gas in expanding from 1 litre to 200 litres at constant temperature against a constant external pressure of 5 atmospheres. [Mechanical equivalent of heat = $4{\cdot}2 \times 10^7$; acceleration due to gravity = 981 cm. per sec. per sec.; 1 atmosphere = 760 mm. of mercury. Specific gravity of mercury = 13·6.]

ANSWERS.

Exercises II. (*Page* 23.)

2. $-6\frac{2}{3}°$. **3.** $-40°$. **4.** 36°, 68°.
5. F 1562, 21·2, − 58; R 680, − 4·8, − 40.
6. C 110, $-2\frac{7}{9}$, $-23\frac{1}{3}$, − 45; R 88, $-2\frac{2}{3}$, $-18\frac{2}{3}$, − 36.
7. C $137\frac{1}{2}$, − 10, $-33\frac{3}{4}$; F $279\frac{1}{2}$, 14, $-28\frac{3}{4}$.

Exercises III. (*Page* 29.)

1. 18·657 ± ·002. **2.** ·644 ± ·001. **3.** Limits of error ± ·0002.
4. 11·0. **5.** 19 sq. cms.
6. − ·7, − ·4, − ·2, − ·7; − ·6, − ·4, − ·3, − ·4.
7. 1·000089. **8.** 1. **9.** 1·0007; ·999961. **10.** 999975; 1·0013.
11. 1·00012; 1·00032. **12.** 100·084; 99·984.

Exercises IV. (*Page* 49.)

5. 1·0057 cub. dec. **8.** 1·1 in. **9.** 100·012 cms.
10. 200·34 cms. **12.** 0·000016. **13.** 0·0432 in.
14. ·03 cm. **15.** 72·2. **17.** 5·485 cub. ft.
18. ·0000258, ·0000086. **20.** 1·0015 pt. **21.** $\frac{d_{100}}{d_{-100}} = ·983$.
22. 10·15 gms. **23.** 2·036 : 1.

Exercises V. (*Page* 73.)

2. 0·0000259. **3.** 1·00029. **4.** 0·000017.
6. 0·0001568. **7.** 0·0000247. **8.** 0·000513.
9. 0·000027. **10.** 0·000029. **11.** 100° C.
12. 0·000155. **13.** $(T - n t c_a)/(1 - n c_a)$. **14.** 0·00091.
17. 20° − 40° C. ·000307.
40° − 60° C. ·000458.
60° − 80° C. ·000596.
80° − 90° C. ·000693.
18. $(1 + \delta)/(1 + \Delta)$. **19.** 761·3 mm., 759·2 mm.

Exercises VI. (*Page* 100.)

3. $\frac{1}{3}$ cub. met. **4.** 2 litres. **5.** 568 mm.
6. 481 c.cms. **7.** 859 c.cms.
8. ·0038. **9.** 661° C.
10. (i) P, T, $\frac{T' V}{T}$; (ii) $\frac{T' P}{T}$, T', V.
11. $\frac{1}{c_r} = 273{\cdot}7$. **12.** ·084 gramme. **13.** 1·0043.
14. 313° C. **15.** 273° C. **16.** − 459° F.
17. ·185 cub. ft. **18.** 345 cub. in. ; 577 cub. in.
19. 534 cc. **20.** 451 cub. in.

Examination Questions. (*Page* 102.)

5. Temperature. **8.** Steam. **9.** Freezing point.
15. Expansion of water is irregular and very small in the neighbourhood of 4° C.
19. ·01 in. **20.** $L_0 = 29{\cdot}996$, $L_{20} = 30{\cdot}007$.
21. ·11 in. **23.** No. **24.** ·00025.
25. 76·14 cms. **26.** 98·6° C. **27.** 763·7 mm.
29. Bulb rises till 6° is reached, then sinks, and at 12° only just floats again. As the temperature rises above 12° the bulb sinks to the bottom.
30. Vessel lifted up 9·5 cms. **31.** 1·5 atmospheres.
32. 2·58 atmospheres.

Exercises VII. (*Page* 112.)

2. 66°. **3.** $\frac{1}{10}$ unit. **4.** 80 units.

Exercises VIII. (*Page* 126.)

7. $11\frac{1}{9}$. **8.** $55\frac{5}{9}$. **9.** 96·8° C.
10. 90·9° C. **11.** 14·2. **12.** ·057.
13. ·214. **14.** 0·64. **15.** 4·79.
16. 756 C. **17.** 25·9° ; 9·1 grammes.
18. 13·2 grammes. **19.** ·090. **20.** ·615.
21. 745°. **22.** 77·3°. **23.** 1·005 ; 1·027 ; 1·016.
24. 12·5°. **25.** 25 : 2 ; $1\frac{23}{27}$° C.
26. 3·41. **27.** 0·46.

EXERCISES IX. (*Page* 147.)

1. The iron. **3.** 40 minutes. **4.** 171·4 calories.
5. 10° C. **8.** 67°. **9.** 44·4 calories.
10. 95·6 pound-degrees. **11.** 78·8. **12.** ·11.
13. 0·033. **14.** ·917. **15.** ·033. **16.** 5·7.

EXAMINATION QUESTIONS. (*Page* 149.)

4. ·47. **5.** ·332. No. **6.** 15 grammes, ·089.

7. Drop the heated salt into turpentine.

8. The liquid has a higher average specific heat, in ratio $\frac{16}{15}$ at the higher temperatures. Yes, very likely.

11. 0·247.

13. Water is above 0° C. and also in motion.

14. (1) Succeed only if body is hot. (2) Succeed only as long as some ice remains.

17. 40 : 61. Probably not true in fact, but data not complete for a more exact answer.

18. $\frac{1}{2}$.

19. Start with ice in a calorimeter at least 10° C. below zero. Then immerse in warm water until temperature of ice gets nearly up to 0° C. Read the fall of temperature of the water and calculate in usual manner.

20. $133\frac{1}{3}$; more required in both cases.

21. 25 grammes. Less if allowed to stand before mixing.

EXERCISES X. (*Page* 162.)

7. 803 mm.

EXERCISES XI. (*Page* 180.)

15. Doubles from 99·93° to 99·96°, and becomes infinitely large at 100° C., *i.e.* it would burst.

16. 742.3 min.

EXERCISES XII. (*Page* 189.)

4. 4·39. **5.** 1·6 lbs. **6.** 22·2° C.
7. 967, 144. **8.** 51° C. **9.** 65·3° C.
10. 537. **11.** 537. **12.** 0·187.

Exercises XIII. (*Page* 203.)

5. The air is full of water vapour. The block of ice cools the air in its vicinity, and hence causes condensation.
14. (1) Lowered more than with water. (2) Scarcely lowered at all.
15. 84 %. 16. 52·9 %. 17. 0° C.
18. 1·205 grammes. 19. − 5° C. (approx.).
20. 9·8° C.; 77·1 %. 21. 73 %.

Examination Questions. (*Page* 205.)

9. Boil, finally probably freeze. 12. 39·8 grammes.
13. 77·6 grammes. 14. ·88 gramme.
15. 5 grammes of liquid boils away. Temperature of mixture is 80°
20. Yes, to a great extent.

Exercises XIV. (*Page* 223.)

4. $5{\cdot}2 \times 10^7$ calories. 5. 1·09. 6. ·0384.
7. $1{\cdot}3 \times 10^7$ calories. 8. 125 calories.
9. ·00013 C.G.S. units.

Exercises XV. (*Page* 231.)

3. In A.
5. (*a*) Convection. (*b*) The air rising from the earth cools by expansion.

Exercises XVI. (*Page* 240.)

2. (*a*) 60 ft.-lbs. (*b*) 7000 ft.-lbs. (*c*) 120 ft.-poundals. (*d*) 10^{11} gr. cms., or 10^6 kilogram-metres. (*e*) 6 ergs.
3. (i) When at the end of the swing. (ii) When in the middle.
4. The bow possesses potential energy due to strain or distortion.
5. (1) 20,000 foot-poundals. (2) 8,000 poundals, 250 pds.-wt. 111,000,000 dynes.

Exercises XVII. (*Page* 257.)

1. The energy of the pumper.
2. Part of the heat is due to the hot gases, the rest due to friction between bullet and barrel.
3. Into heat.
4. Mechanical energy is turned into heat energy.

5. During evaporation and the uplifting of clouds, heat energy becomes potential and kinetic energy. When the rain reaches the earth again all the potential energy and nearly all the kinetic energy has disappeared.

6. Chemical energy in coal→Heat energy of flames and water →Kinetic energy of train and passengers→Kinetic energy of air and heat energy of rails and rubbing parts of the wheels.

7. By a specific heat process. **9.** 700 ft.-pds., $2{\cdot}09 \times 10^7$ ergs.

10. 1° C., 1·8° F. **11.** 165 pounds.

12. 112,000 ft.-pds., 335×10^7 ergs,
752,000 ft.-pds., 2250×10^7 ergs.

13. 1,390 foot-pounds. **14.** One-thirteenth.

Exercises XVIII. (*Page* 286.)

10. 13%. **11.** 2·56%.

Examination Questions. (*Page* 287.)

2. 1·8 calories. **3.** 5,260 grammes.

4. No ; hotter on the copper rod. No ; temperature of polished rod is above that of blackened rod. Bore small holes, half filled with mercury, and insert bulbs of thermometers in them.

8. 1800 calories.

16. (1) Rise faster in blackened vessel.
(2) Cool faster in the blackened vessel.

22. 34,000 metres.

23. 420 gm.-metres per calorie.

25. 75 watts.

26. 22,500 cm. per sec.

27. 4·8 calories.

INDEX.

The numbers refer to pages.

Absolute conductivity, 219.
,, expansion of mercury, 61.
,, temperature, 96.
Absorption, coefficient of, 273
Acceleration, 232.
Adiathermanous, 261.
Apparent expansion, 53.

Barometer, correction for temperature, 72.
Black's ice calorimeter, 143.
Boiling, or ebullition, 164, 170.
Boiling point, 169.
,, ,, , determination of, 174.
,, ,, , influence of pressure on, 166.
Boyle's Law, 79.
Breezes, land and sea, 228.
Breguet's thermometer, 47.
Bunsen's ice calorimeter, 145.

Caloric, 242.
Calorie, 107.
Calorimeter, 109.
,, , *Bunsen's* ice, 145.
,, , *Joly's* steam, 187.
Celsius, 15.
Centigrade scale, 15.
Change of state, 6.
,, ,, zero, 21.
Charles's law of the expansion of gases, 89.
Chronometer balance wheel, 45.
Cloud, 200.
Combustion, 6.
Condensation, 163, 164, 178.
Conductivity for heat, 208.
,, , absolute thermal, 216.
,, , absolute thermal, determination of, 219.
,, , relative thermal, determination of, 221.
Convection currents, 225.
Cooling correction in calorimetry, 120.
Cryophorus, *Wollaston's*, 178.

Dalton's law of expansion of gases, 89, 97.
,, laws of vapours, 157, 159.
Davy lamp, 211.
Davy's experiments, 244.
Density, and temperature, 48.
De Saussure's hygroscope, 199.
Dew, formation of, 202.
Dew-point, 193.
Diathermancy, 261.
Dines's hygrometer, 197.
Distillation, 178.
Dulong and Petit, on expansion of mercury, 61.
Dynamical theory of heat, 242 *et seq.*

Ebullition, 164, 170.
,, , laws of, 168.
Effects of heat, 4.
Emission, coefficient of, 273.
Emissive powers, 273.
Energy, 236, 237, 250.

Energy, heat a form of, 3, 242, 251.
,, , potential, 237.
,, , several forms of, 240.
Evaporation, 152, 175.
,, , cold caused by, 177.
,, , conditions favourable to, 176.
Expansion, compensation for, 43.
,, in freezing, 132.
,, of gases, 5, 76.
,, of liquids, 5, 52.
,, of mercury, 61.
,, of solids, 5, 30.
,, of water, 67.
,, , practical effects of, 42, 46.
,, , real and apparent, 53.
,, , relation between linear and volume, 35.

Fahrenheit's scale, 15.
Fog, 201.
Force of expansion, 46.
,, , units of, 233.
Franklin's experiment, 166.
Freezing mixture, 142.
Freezing point, influence of pressure on, 137.

Gas thermometer, 93.
Gay Lussac on the expansion of gases, 83.
Graham's mercurial pendulum, 45.
Gridiron pendulum, *Harrison's*, 43.

Hail, 201.
Harrison's gridiron pendulum, 43.
Heat, a form of energy, 3, 242, 250.
,, , effects of, 4.
,, , nature of, 3, 146.
,, , transference of, 208.
,, , unit of, 106.
Hope's experiment, 71.
Humidity, relative, 192.
Hygrometers, 195.
Hygrometric state, 192.
Hygroscopes, 198.
Hypsometer, 173.

Ingenhousz's apparatus, 221.
Invar, 46.
Inverse squares, law of, 265.
Isotropic body, 31.

Joly's steam calorimeter, 187.
Joule, the, 250.
Joule's equivalent, 250.
,, experiments, 245, 253.

Kathetometer, 63.

Latent heat, 138, 182.
,, ,, , nature of, 146.
,, ,, of fusion of ice, 139
,, ,, of vaporisation of water, 183.
Leslie's cube, 264.
,, experiment, 177.
,, thermoscope, 100, 255.
Liebig's condenser, 165.
Linear expansion, coefficient of, 32.
Liquefaction or fusion, 130.

Mason's hygrometer, 198.
Mass, 232.
Matter, 232.
Maximum and minimum thermometers, 18.
Melting point, determination of, 134.
,, ,, , effect of pressure on, 137.
Mercury, real expansion of, 61.
,, thermometer, 11.
Micrometers, 37.
Mist, 201.
Momentum, 232.

Nature of heat, 3, 146.
Newton's law of cooling, 272.
Norwegian cooking box, 211.

Papin's digester, 172.
Pendulum, *Graham's*, 45.
,, , *Harrison's*, 43.
Power, units of, 236.
Pressure, influence of on boiling points, 166.
,, , influence of on melting points, 137.
Pressures, gaseous, 92, 98.
Prevost's theory of exchanges, 278.

Radiant heat, 272, 274.
,, ,, , reflection of, 266.
,, ,, , refraction of, 269.
,, ,, , transmission of, 184, 264, 191.
Radiating power, 273, 277.
Radiation, nature of, 259.
Rain, 201.
Real expansion, 53.
Réaumur's scale, 15.
Regelation, 137.
Regnault on expansion of gases, 87.
,, on expansion of mercury, 64.
,, on specific heat, 119.
Regnault's hygrometer, 195.
Roy and Ramsden on expansion of metals, 39.
Rumford's experiments, 243.

Saturated vapour, 160.
Significant figures, 26.
Solidification, or freezing, 130.
Solution, 141.
Soret's formula, 173.
Specific heat, 115.
,, ,, , determination of by cooling, 121.
,, ,, , determination of by mixture, 116.
Stationary state of conduction, 213.
Steam calorimeter, 187.
Sublimation, 179.
Surface expansion, coefficient of, 32.

Tables :—
Boiling points 174.
Coefficients of expansion of liquids, 67.
Coefficients of linear expansion of solids, 41.
Important temperatures, 23.
Joule's equivalent, 250.
Latent heats of fusion, 141.
Latent heats of vaporisation, 187.
Melting points. 137.
Specific heats, 125.
Thermal conductivities, 223.
Vapour pressure of mercury, 159.
Vapour pressure of water, 156, 195.
Volume and temperature of water-stuff, 189.
Volume, density, and temperature of water, 70.
Temperature, 2.
,, , absolute, 96.
,, , critical, 162.
,, gradient, 214.
Temperatures, some important, 23.
Thermal capacity, 113.
,, conductivity, 216.
,, ,, , determination of, 219.
Thermal conductivity, relative, 221.
Thermo-dynamics, first law of, 250.
Thermometer, 8.
,, , air or gas, 20, 93.
,, , alcohol, 18.
,, , clinical, 20.
,, , maximum, 18.
,, , mercury, 11.
,, , mercury, fixed points of, 13.
,, , mercury, graduation of, 15.
,, , metallic, 47.
,, , minimum, 18.
,, , *Phillip's*, 20.

Thermometer, *Rutherford's*, 19.
,, , *Six's*, 18.
,, , weight, 56.
Thermometric substance, 124.
Thermopile, 263.
Thermoscope, 100, 263.
Trade winds, 229.
Transference of heat, 208.

Unit of heat, 106.
Unsaturated vapour, 160.

Vacuum vessels, 286.
Vaporisation, 152, 163.
Vapour, compared with a gas, 162.
,, pressure, 152.
,, ,, of water, 154, 167.
,, , saturated and unsaturated, 160.
Velocity, 232.
Ventilation, 227.
Volume dilatometer, 59.
,, ,, , constant, 60.
Volume expansion, coefficient of, 34.

Water equivalent, 110.
,, , expansion of, 67.
,, , ,, , on boiling, 189.
,, , ,, , on freezing, 132.
,, vapour in atmosphere, 191.
Weight, 233.
,, thermometer, the, 56.
Wells' theory of the formation of dew, 202.
Wet and dry bulb hygrometer. 198.
Winds, 228.
,, , Trade, 229.
Wollaston's cryophorus, 178.
Work, 234.
,, done by a gas, 252.
,, , units of, 235.

Zero, absolute, 96.
,, , change of, 21.

PRINTED AT THE BURLINGTON PRESS, CAMBRIDGE.

BINDING LIST AUG 17 1951

Phys
Thermodyn
S

Stewart, Robert Wallace
Senior heat.

129445

University of Toronto
Library

DO NOT
REMOVE
THE
CARD
FROM
THIS
POCKET

Acme Library Card Pocket
LOWE-MARTIN CO. LIMITED